

IMPERIAL

Understanding Aggregated Flexibility in Active Distribution Networks

Game on! Seminar

KTH Royal Institute of Technology, Stockholm

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A few words about me

Moscow → Manchester → London

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Power systems

Electricity markets

Mathematical optimisation

Game theory

Data valuation

Machine learning

Forecasting

Agenda for today's talk

Flexibility aggregation in active distribution networks:

- ◆ Intuition and principles
- ◆ Models and assumptions
- ◆ Results and challenges

Intuition

Where are distributed energy resources (DER)?

Example of a suburban distribution system: Luton, England, UK



Typical household demand: 0.5 kW average, 3-10 kW peak
Total capacity of a low-voltage distribution network: from 0.5 MW to tens of MW

Intuition

Where are distributed energy resources (DER)?

→ Controllable renewable energy sources

Rooftop solar systems



Capacity: 2-6 kW

Small solar power plants



Capacity: from 50 kW to 5 MW

Intuition

Where are distributed energy resources (DER)?

→ Battery energy storage systems (BESS)

→ Flexible demand

Small-scale residential and commercial BESS



Capacity: from 5 kWh to 1-2 MWh

EV charging infrastructure



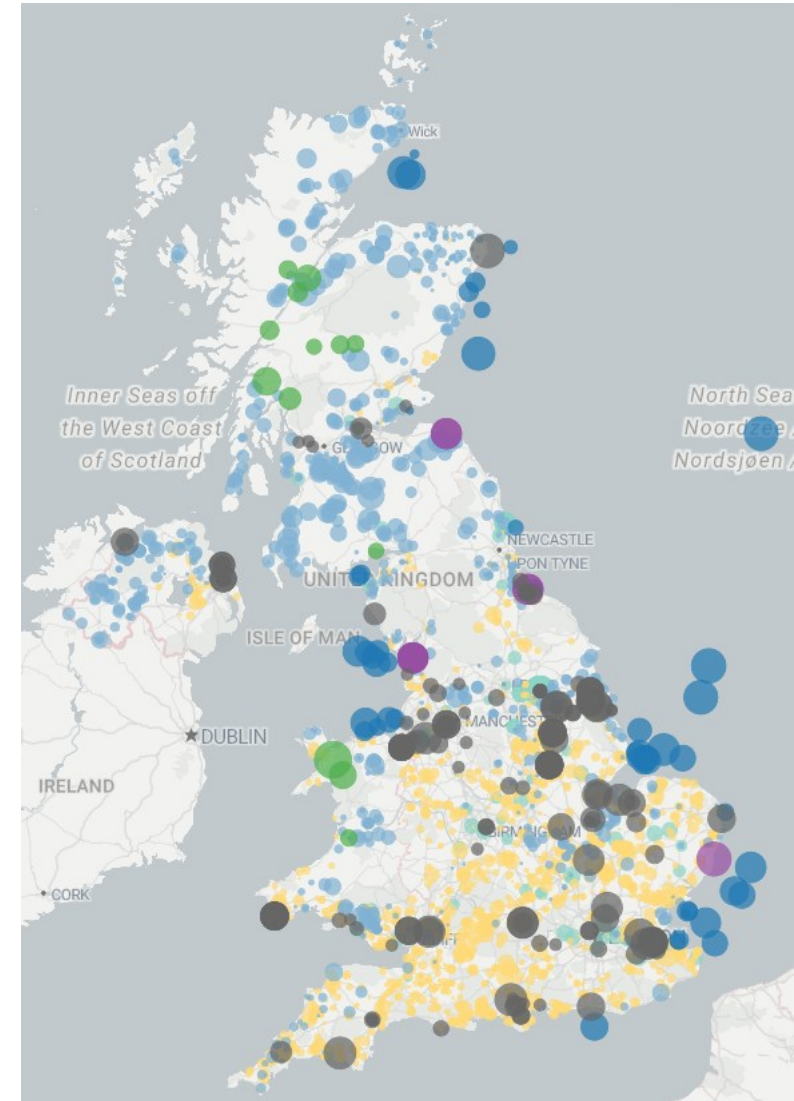
Rated power: from 7 kW to 150+ kW

Intuition

The scale of DER integration in the UK:

- <https://gb.asglab.uk/> developed by Dawei Qiu (University of Exeter)
- <https://www.energydashboard.co.uk/map>

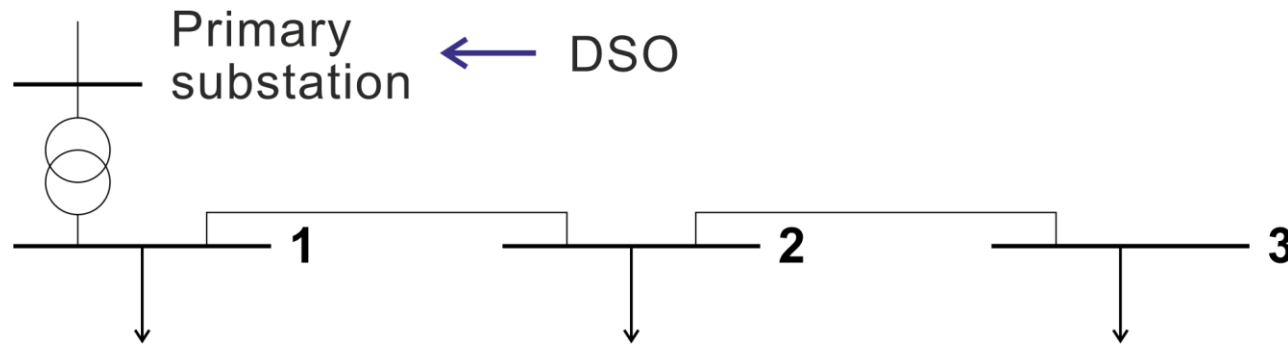
- There are already hundreds of small-scale solar and wind power plants, and thousands of EV charging stations.
- Distribution system operators (DSO) need to know how these DERs can be controlled and coordinated, and how to model their impact on distribution networks.



Intuition

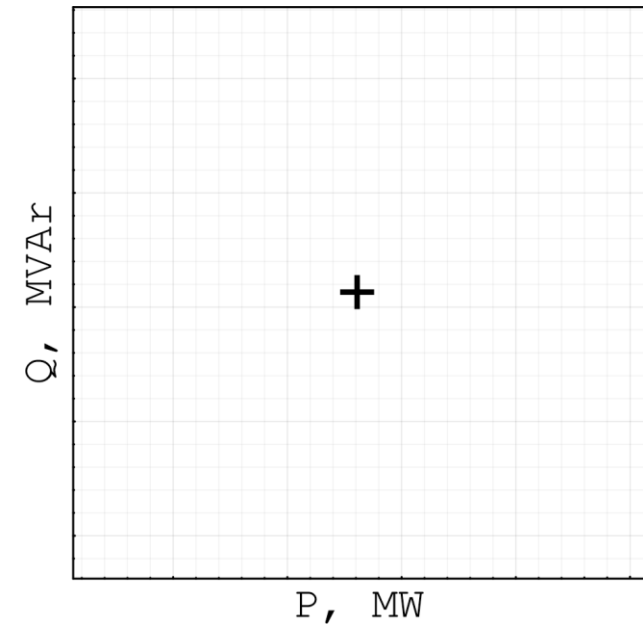
What is flexibility?

Example: traditional (passive) distribution network



No flexible DERs are operating in this network.

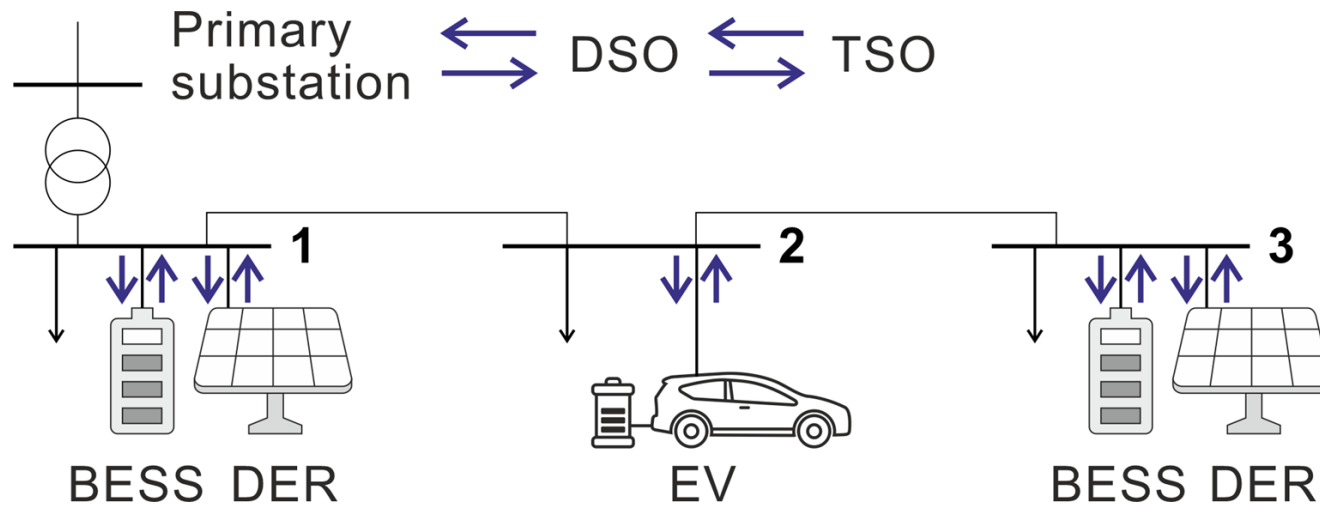
Total network power consumption:



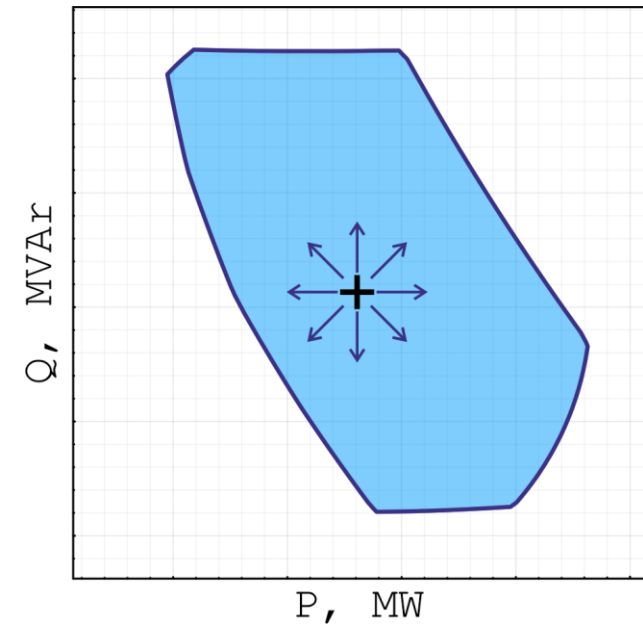
Intuition

What is flexibility?

Example: flexible active distribution network (ADN) with DER



Total network power consumption:



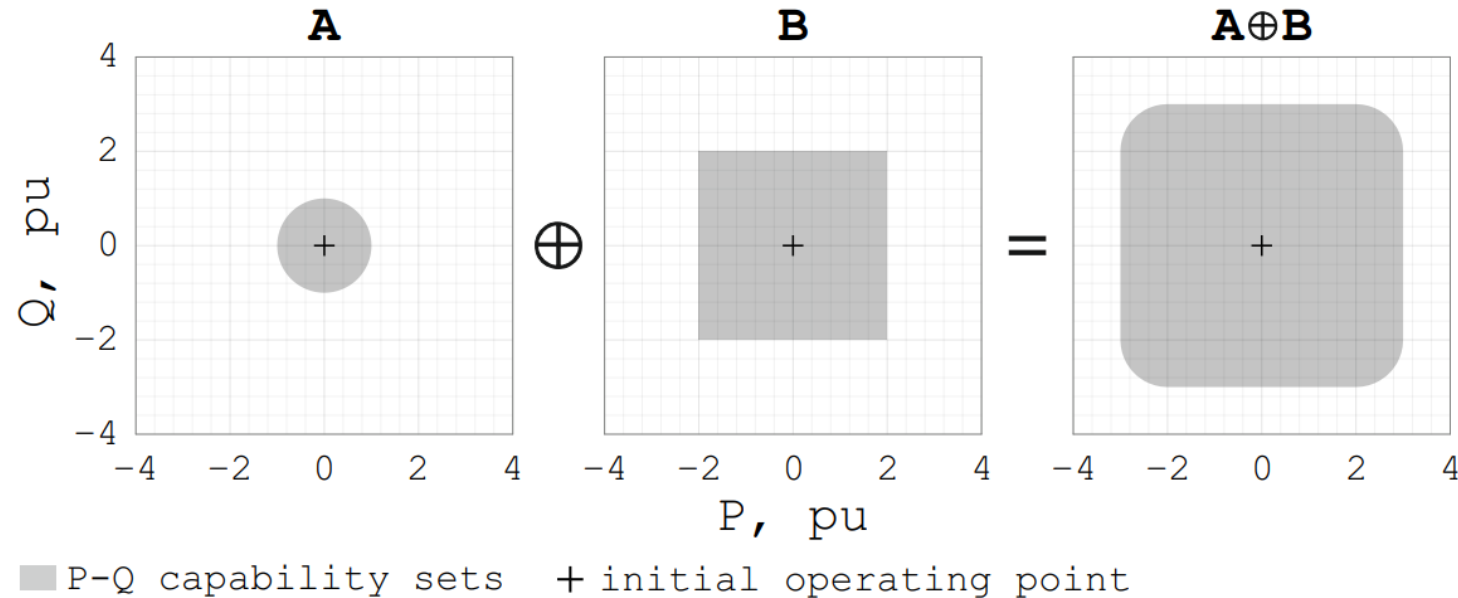
Flexible units are resources with the technical ability to adjust their power exchange with the grid.

Aggregated DER flexibility of ADNs can be characterised as sets of feasible operating points at the TSO-DSO interface. Such sets are known as the **P-Q flexibility areas** or **nodal operating envelopes**.

Flexibility aggregation modelling

Operation limits of multiple flexible units can be mathematically described as the Minkowski addition of their P-Q capability sets:

$$A \oplus B = \{a + b \mid a \in A, b \in B\}$$



However, the Minkowski addition does not account for any physical limits (e.g., network constraints).

Therefore, more advanced optimization-based models are used for flexibility aggregation.

Flexibility aggregation modelling

Nonlinear exact AC optimal power flow (OPF) formulation for radial distribution networks:

Variables:

$$\begin{aligned}
 p_{n,g}, q_{n,g} & n \in \mathcal{N}, g \in \mathcal{G} \\
 p_{nm}, q_{nm} & (n, m) \in \mathcal{L} \\
 V_n \quad (V_n^2 = w_n) & n \in \mathcal{N} \\
 i_{nm} \quad (i_{nm}^2 = l_{nm}) & (n, m) \in \mathcal{L} \\
 p_{n,f}^\uparrow, p_{n,f}^\downarrow, q_{n,f}^\uparrow, q_{n,f}^\downarrow & n \in \mathcal{N}, f \in \mathcal{F} \\
 x_{n,f} \in \{0, 1\} & n \in \mathcal{N}, f \in \mathcal{F} \\
 x_{n,f}^{p^\uparrow}, x_{n,f}^{p^\downarrow}, x_{n,f}^{q^\uparrow}, x_{n,f}^{q^\downarrow} \in \{0, 1\} & n \in \mathcal{N}, f \in \mathcal{F} \\
 \mathfrak{r}^{p^\uparrow}, \mathfrak{r}^{p^\downarrow}, \mathfrak{r}^{q^\uparrow}, \mathfrak{r}^{q^\downarrow} \in \{0, 1\} & n \in \mathcal{N}, f \in \mathcal{F}
 \end{aligned}$$

Flexible power

Binary variables of flexible units' activation

Constraints:

$$p_{n,g}^{\min} \leq p_{n,g} \leq p_{n,g}^{\max} \quad \forall n \in \mathcal{N}, g \in \mathcal{G} \quad (2a)$$

$$q_{n,g}^{\min} \leq q_{n,g} \leq q_{n,g}^{\max} \quad \forall n \in \mathcal{N}, g \in \mathcal{G} \quad (2b)$$

$$p_{nm} = \sum_{d \in \mathcal{D}, g \in \mathcal{G}} (p_{m,d} - p_{m,g}) \quad (2c)$$

$$+ \sum_{f \in \mathcal{F}} (x_{m,f}^{p^\downarrow} p_{m,f}^\downarrow - x_{m,f}^{p^\uparrow} p_{m,f}^\uparrow)$$

$$+ \Re(Z_{nm}) l_{nm} + \sum_{(m,k) \in \mathcal{L}} p_{mk} \quad \forall (n, m) \in \mathcal{L}$$

$$q_{nm} = \sum_{d \in \mathcal{D}, g \in \mathcal{G}} (q_{m,d} - q_{m,g}) \quad (2d)$$

$$+ \sum_{f \in \mathcal{F}} (x_{m,f}^{q^\downarrow} q_{m,f}^\downarrow - x_{m,f}^{q^\uparrow} q_{m,f}^\uparrow)$$

$$+ \Im(Z_{nm}) l_{nm} + \sum_{(m,k) \in \mathcal{L}} q_{mk} \quad \forall (n, m) \in \mathcal{L}$$

$$w_m = w_n + |Z_{nm}|^2 l_{nm} \quad (2e)$$

$$- 2(\Re(Z_{nm}) p_{nm} + \Im(Z_{nm}) q_{nm}) \quad \forall (n, m) \in \mathcal{L}$$

$$p_{nm}^2 + q_{nm}^2 = l_{nm} w_n \quad \forall (n, m) \in \mathcal{L} \quad (2f)$$

$$p_{nm}^2 + q_{nm}^2 \leq (\mathcal{S}_{nm}^{\max})^2 \quad \forall (n, m) \in \mathcal{L} \quad (2g)$$

$$(V_n^{\min})^2 \leq w_n \leq (V_n^{\max})^2 \quad \forall n \in \mathcal{N} \quad (2h)$$

$$0 \leq p_{n,f}^\uparrow \leq \mathfrak{r}^{p^\uparrow} x_{n,f}^{p^\uparrow} p_{n,f}^{\uparrow \max} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \quad (2i)$$

$$0 \leq p_{n,f}^\downarrow \leq \mathfrak{r}^{p^\downarrow} x_{n,f}^{p^\downarrow} p_{n,f}^{\downarrow \max} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \quad (2j)$$

$$0 \leq q_{n,f}^\uparrow \leq \mathfrak{r}^{q^\uparrow} x_{n,f}^{q^\uparrow} q_{n,f}^{\uparrow \max} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \quad (2k)$$

$$0 \leq q_{n,f}^\downarrow \leq \mathfrak{r}^{q^\downarrow} x_{n,f}^{q^\downarrow} q_{n,f}^{\downarrow \max} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \quad (2l)$$

$$x_{n,f}^{p^\uparrow} + x_{n,f}^{p^\downarrow} \leq x_{n,f} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \quad (2m)$$

$$x_{n,f}^{q^\uparrow} + x_{n,f}^{q^\downarrow} \leq x_{n,f} \quad \forall n \in \mathcal{N}, f \in \mathcal{F} \quad (2n)$$

Objective function = maximisation of flexibility provision

The inclusion of binary variables leads to a combinatorial OPF problem – mixed integer quadratically constrained programming (MIQCP).

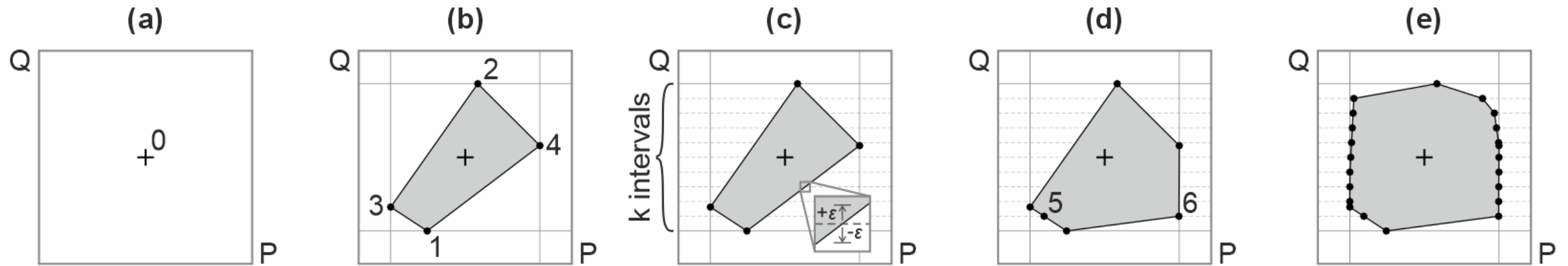
The MIQCP model was implemented in JuMP for Julia programming language and solved with the Gurobi solver.



Flexibility aggregation modelling

How to find the P-Q flexibility area using the OPF model?

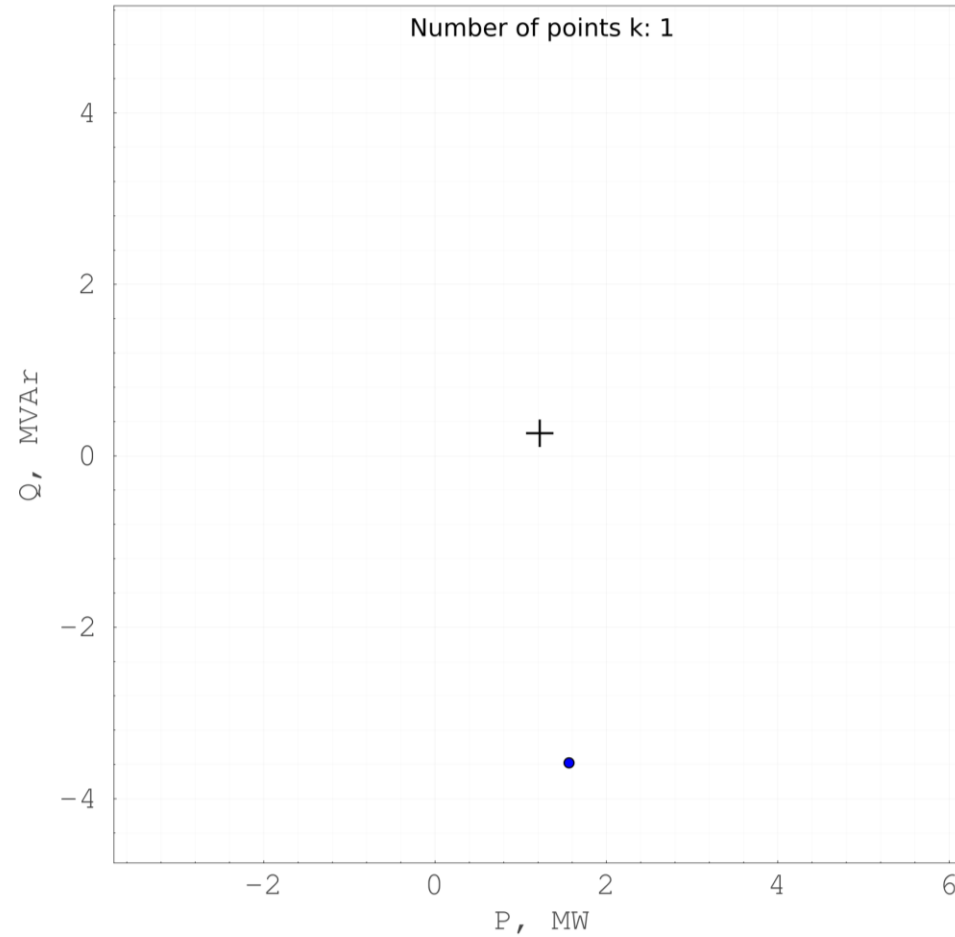
Iterative estimation of the P-Q flexibility area boundary via the “ ϵ -constraint method”:



Flexibility aggregation modelling

How to find the P-Q flexibility area using the OPF model?

Iterative estimation of the P-Q flexibility area boundary via the “ ϵ -constraint method”:



Flexibility aggregation modelling

Brief intro in **cooperative game theory**:

Coordination of flexible units is a combinatorial problem with 2^N possible combinations of units.

In game-theoretic terms, we can refer to units as **players**, and their combination as **coalitions**.

A cooperative game with 4 players has $2^4=16$ coalitions. A game with 10 players has $2^{10}=1024$ coalitions.

Marginal contribution:

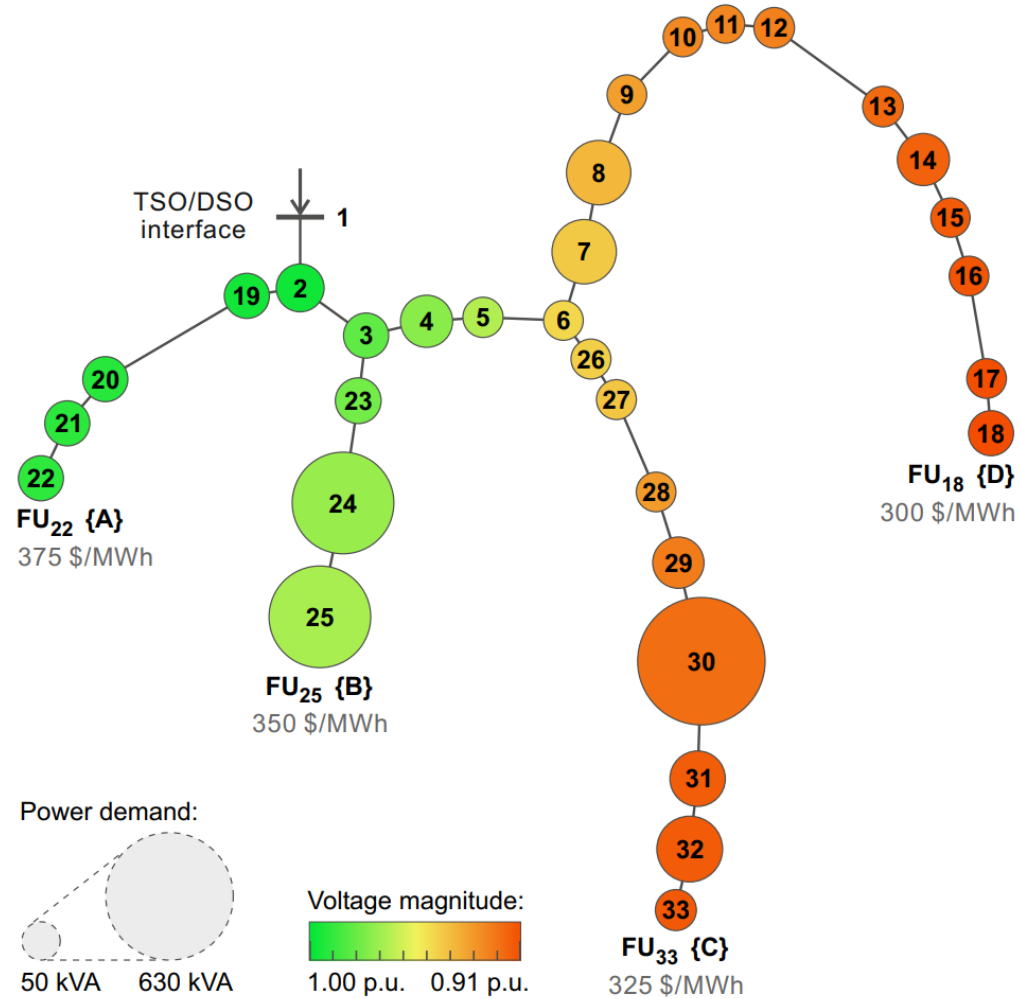
$$MC(S)_i = v(S \cup \{i\}) - v(S) \quad \forall i \in S \quad \forall S \subseteq N$$

The Shapley value:

$$Sh_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (|N| - |S| - 1)!}{|N|!} MC(S)_i$$

Flexibility aggregation modelling

Case study: 33-bus radial distribution network with 4 flexible units

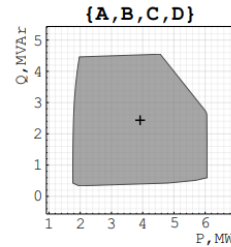


Flexibility aggregation modelling

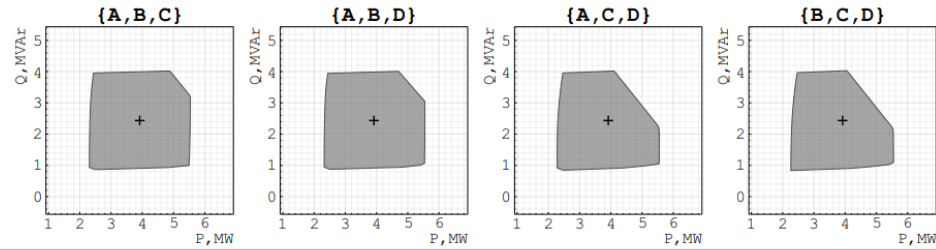
Aggregated P-Q flexibility

Coalitional analysis

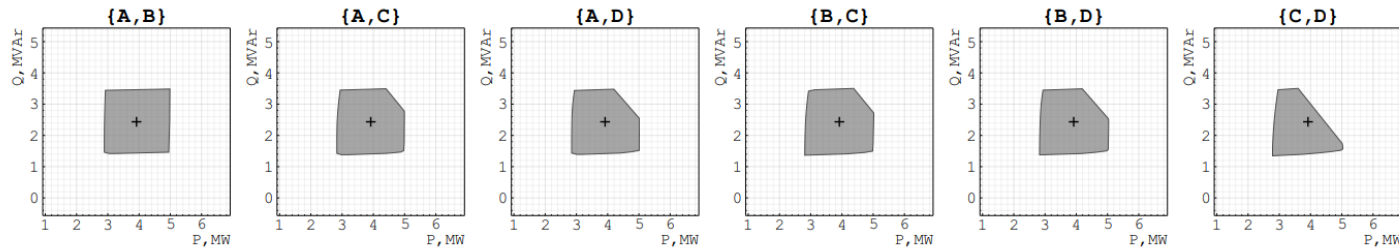
The grand coalition:



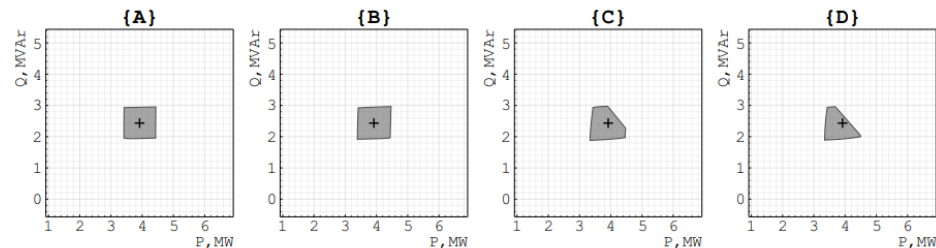
Coalitions of three flexible units:



Coalitions of two flexible units:



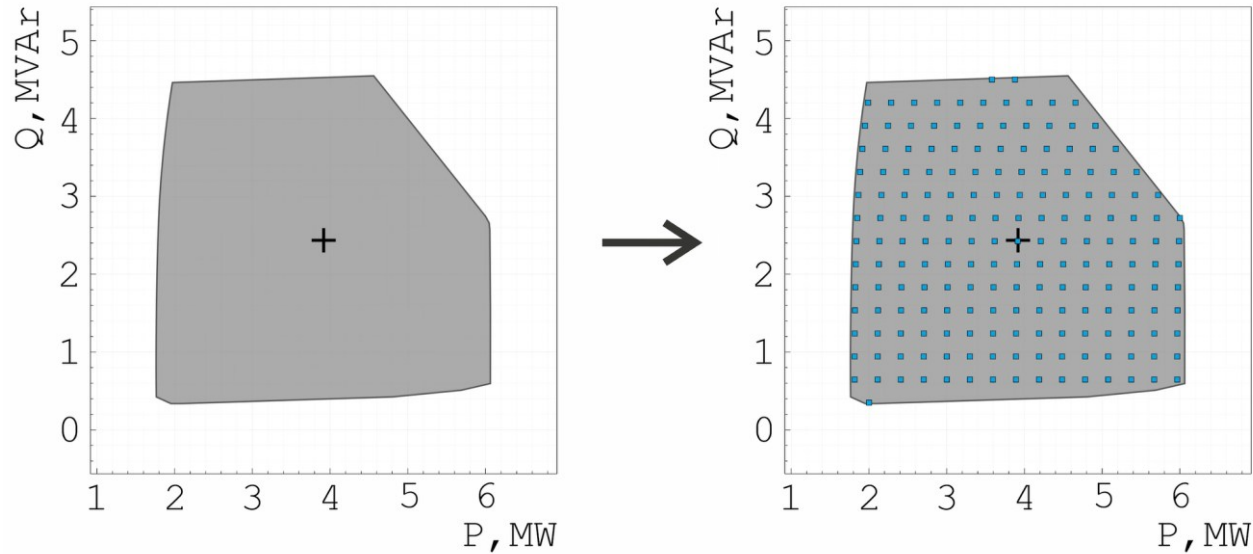
Coalitions of one flexible unit:



Flexibility aggregation modelling

How to optimally dispatch flexible units within the identified limits?

→ We can discretise the feasible space and solve the cost-minimisation dispatch problem



MODEL Cost minimisation of flexibility requests [MIQCP]

Objective:

$$\min \sum_{n \in \mathcal{N}} \sum_{f \in \mathcal{F}} C_{n,f}(p_{n,f}^{\uparrow}, p_{n,f}^{\downarrow}, q_{n,f}^{\uparrow}, q_{n,f}^{\downarrow}) \quad (5a)$$

Constraints:

$$\text{modified DistFlow model (2a)-(2n)} \quad (5b)$$

$$\text{aggregated flexibility } P_n, Q_n \text{ (3a)-(3b)} \quad n = n^{\text{ref}} \quad (5c)$$

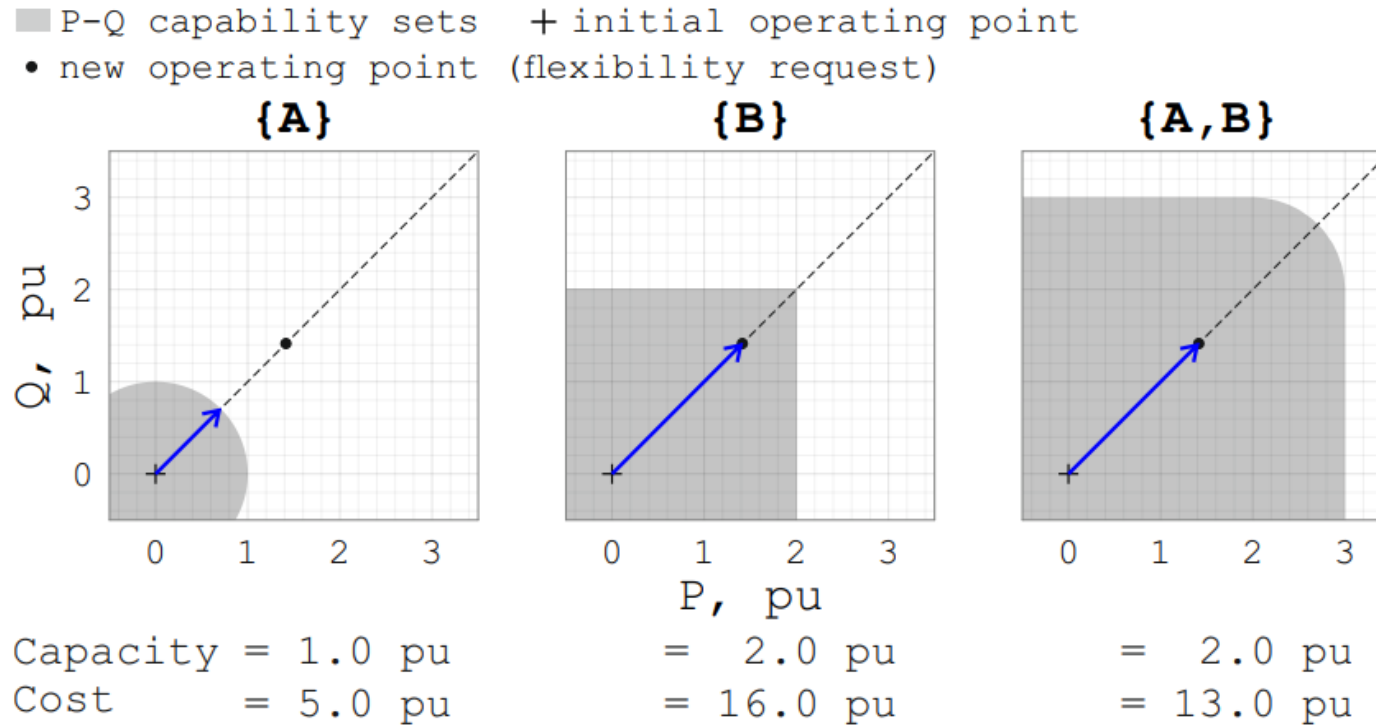
$$P_n = P'_n \quad n = n^{\text{ref}} \quad (5d)$$

$$Q_n = Q'_n \quad n = n^{\text{ref}} \quad (5e)$$

Flexibility aggregation modelling

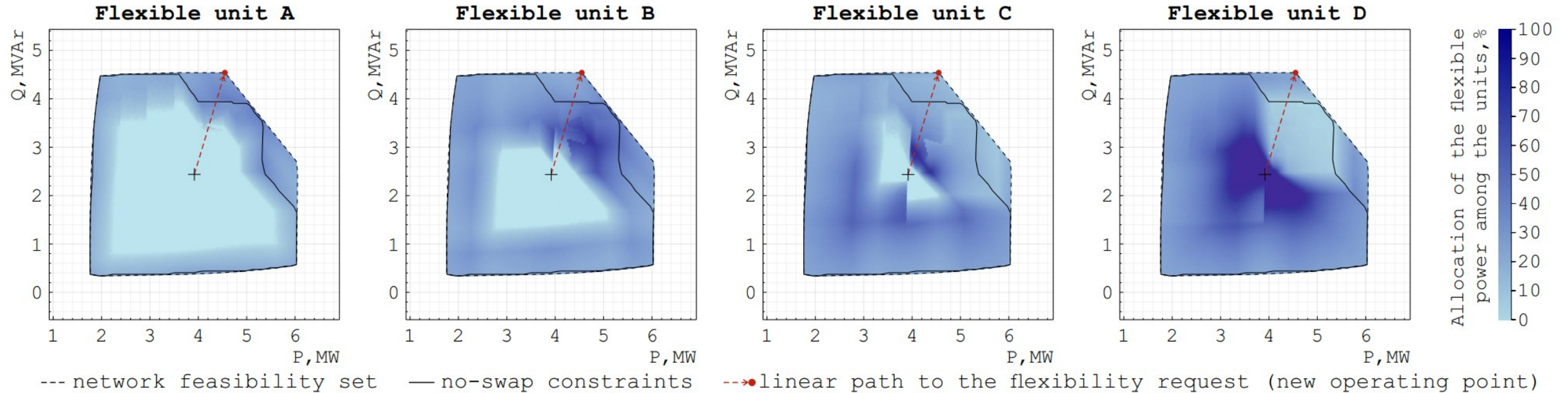
How to optimally dispatch flexible units within the identified limits?

→ We can discretise the feasible space and solve the cost-minimisation dispatch problem



Results and challenges

Allocation of flexible power among the units according to the cost-minimising dispatch (15,000 operating points):



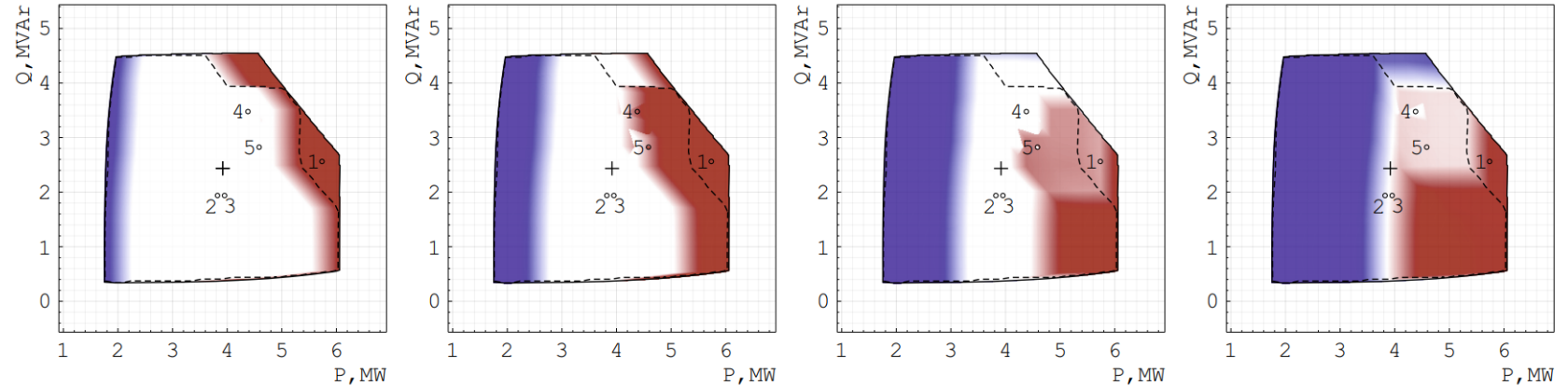
Q: Why are the dispatch solutions so complex?

Results and challenges

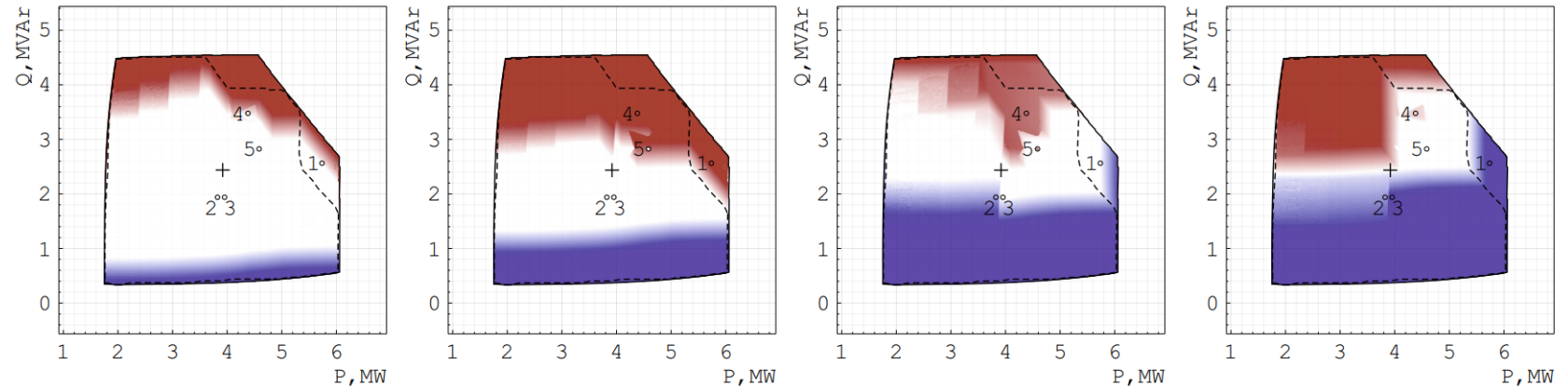
Let's take a closer look at the optimal dispatch solutions:

Flexible unit A (bus 22) Flexible unit B (bus 25) Flexible unit C (bus 33) Flexible unit D (bus 18)

Flexible active power management, MW:

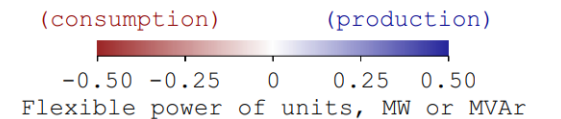


Flexible reactive power management, MVAR:



Challenge:
Flexible power swaps between units,
which is not practically possible without
strong coordination

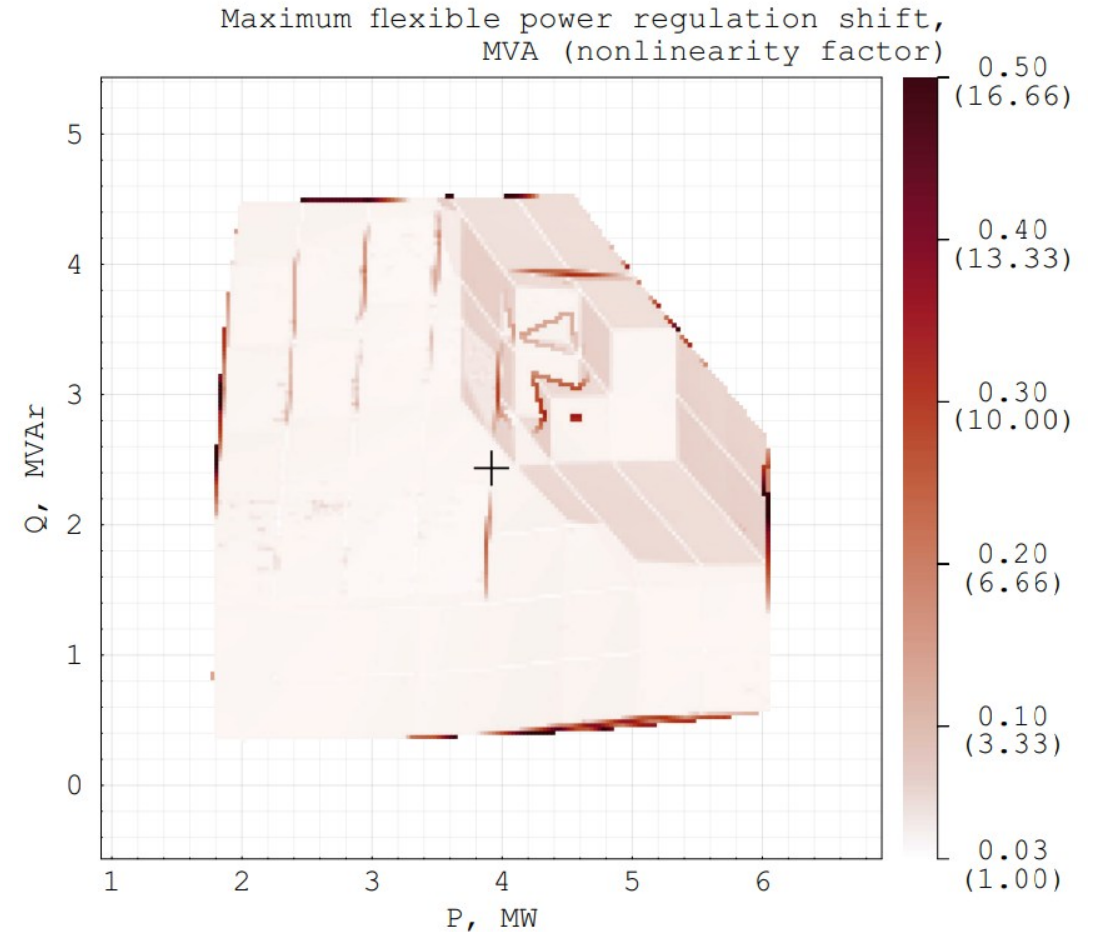
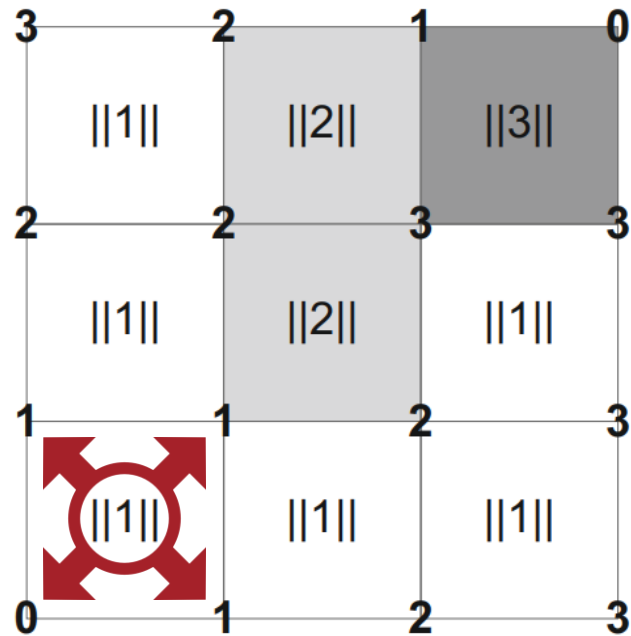
- + Initial operating point
- Network feasibility set
- Feasibility set without flexible power swap between the units
- Operating points of interest



Results and challenges

How to assess nonlinearity of the optimal unit dispatch?

Example: nonlinearity assessment algorithm for a 16-point grid

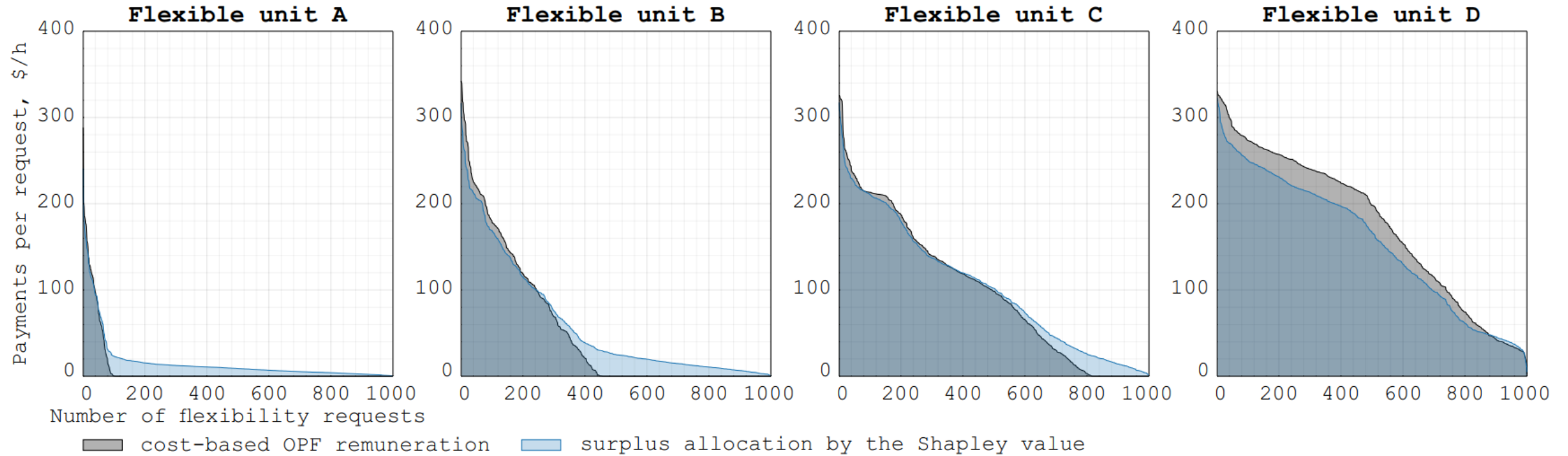


Challenge: rapid nonlinear changes in flexible units dispatch

Results and challenges

How to price flexibility services and pay to flexible units?

→ Economic dispatch (merit order payments) vs Shapley-based allocations:

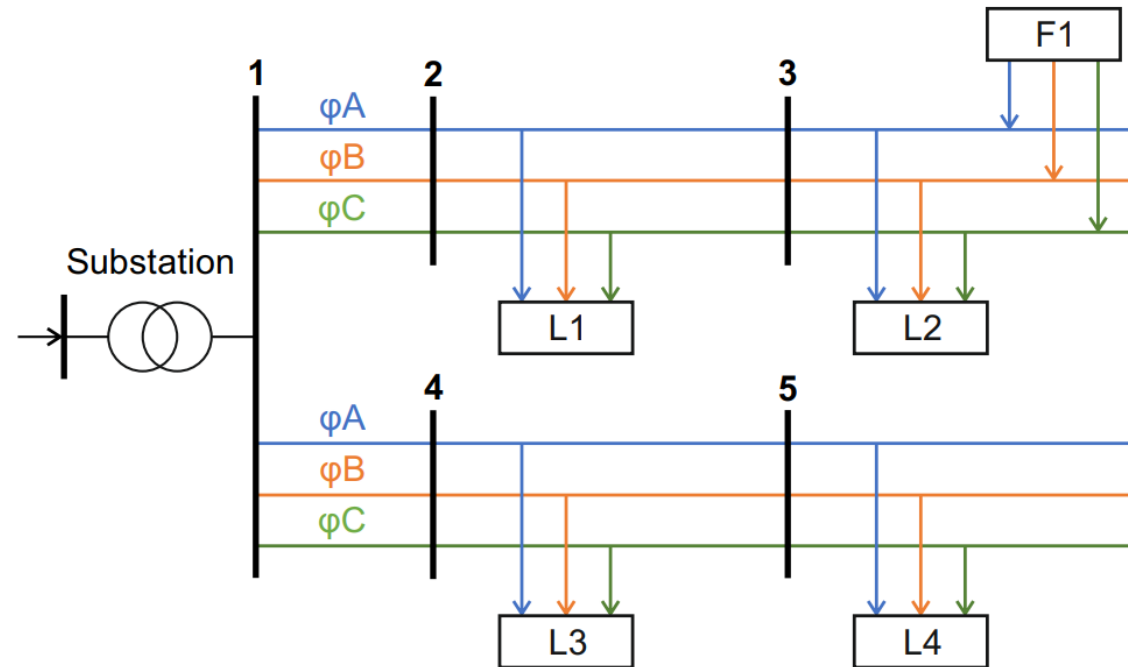


Shapley value sees the usefulness of more expensive units (and pays for availability)

Results and challenges

How to analyse aggregated flexibility in low-voltage unbalanced networks?

→ Multi-phase flexibility aggregation models are needed



Results and challenges

This becomes an even more complex problem:

The model has many more variables

Variables: (for each phase $\phi \in \{a, b, c\}$)

$$\begin{aligned}
 V_{i,\phi} & i \in \mathcal{N} \\
 s_{i,\phi} & i \in \mathcal{N} \\
 s_{ij,\phi} & (i,j) \in \mathcal{L} \\
 P_{g,i,\phi}, Q_{g,i,\phi} & i \in \mathcal{N}, g \in \mathcal{G} \\
 P_{f,i,\phi}, Q_{f,i,\phi} & i \in \mathcal{N}, f \in \mathcal{F} \\
 V_{1i}, V_{2i}, VUF_i & i \in \mathcal{N}^{\text{vu}}
 \end{aligned}$$

It allows to explicitly introduce voltage unbalance and phase coordination

AC OPF constraints:

$$V^{\min} \leq |V_{i,\phi}| \leq V^{\max} \quad \forall i \in \mathcal{N} \quad (1b)$$

$$s_{ij,\phi} = \text{diag} \left[V_{i,\phi} (V_{i,\phi} - V_{j,\phi})^H Y_{ij}^H \right] \quad \forall (i,j) \in \mathcal{L} \quad (1c)$$

$$s_{i,\phi} = \sum_{(i,j) \in \mathcal{L}} s_{ij,\phi} \quad \forall i \in \mathcal{N} \quad (1d)$$

$$\sum_{g,d,f} (P_{g,i,\phi} - P_{d,i,\phi} + P_{f,i,\phi}) = \Re(s_{i,\phi}) \quad \forall i \in \mathcal{N} \quad (1e)$$

$$\sum_{g,d,f} (Q_{g,i,\phi} - Q_{d,i,\phi} + Q_{f,i,\phi}) = \Im(s_{i,\phi}) \quad \forall i \in \mathcal{N} \quad (1f)$$

$$|s_{ij,\phi}| \leq s_{ij,\phi}^{\max} \quad \forall (i,j) \in \mathcal{L} \quad (1g)$$

$$P_{g,i,\phi}^{\min} \leq P_{g,i,\phi} \leq P_{g,i,\phi}^{\max} \quad \forall g \in \mathcal{G} \quad (1h)$$

$$Q_{g,i,\phi}^{\min} \leq Q_{g,i,\phi} \leq Q_{g,i,\phi}^{\max} \quad \forall g \in \mathcal{G} \quad (1i)$$

$$P_{f,i,\phi}^{\min} \leq P_{f,i,\phi} \leq P_{f,i,\phi}^{\max} \quad \forall f \in \mathcal{F} \quad (1j)$$

$$Q_{f,i,\phi}^{\min} \leq Q_{f,i,\phi} \leq Q_{f,i,\phi}^{\max} \quad \forall f \in \mathcal{F} \quad (1k)$$

Voltage unbalance limits:

$$V_{1i} = \frac{V_{i,\{a\}} + aV_{i,\{b\}} + a^2V_{i,\{c\}}}{3} \quad i \in \mathcal{N}^{\text{vu}} \quad (1l)$$

$$V_{2i} = \frac{V_{i,\{a\}} + a^2V_{i,\{b\}} + aV_{i,\{c\}}}{3} \quad i \in \mathcal{N}^{\text{vu}} \quad (1m)$$

$$VUF_i = \frac{V_{2i}}{V_{1i}} \times 100\% \quad i \in \mathcal{N}^{\text{vu}} \quad (1n)$$

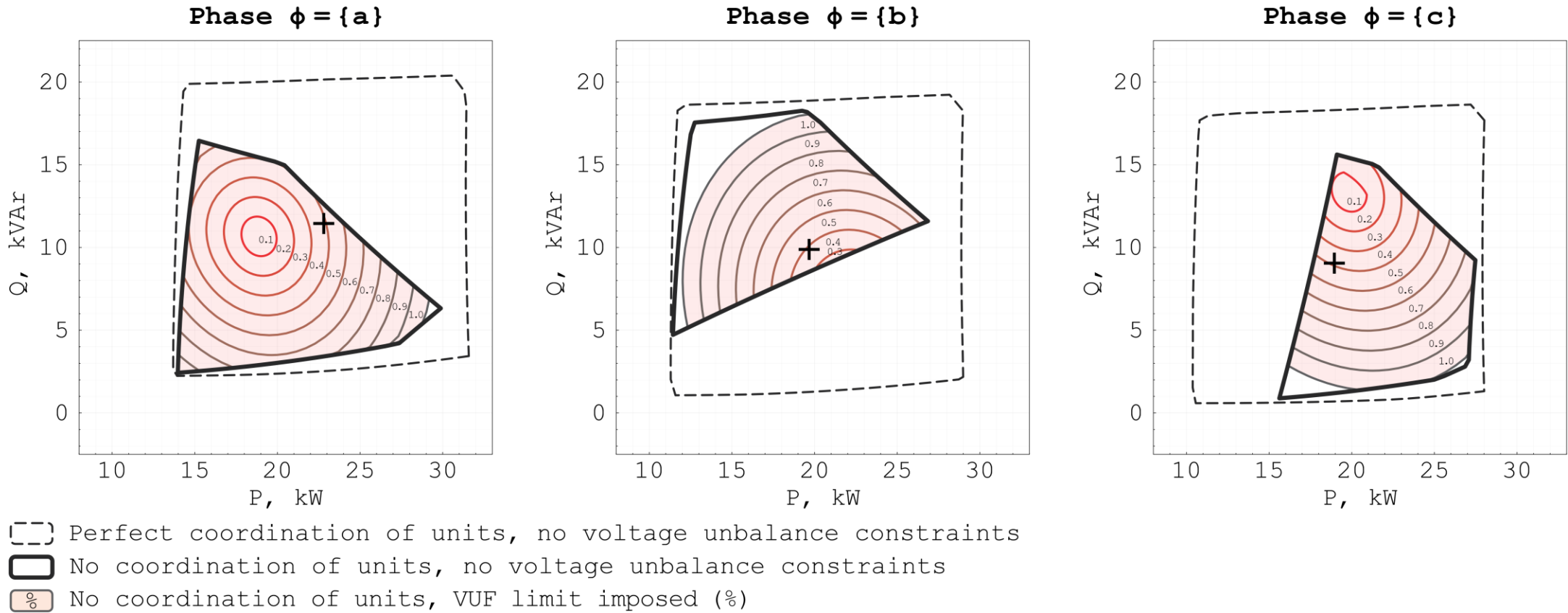
$$VUF_i \leq \overline{VUF} \quad i \in \mathcal{N}^{\text{vu}} \quad (1o)$$

Phase coordination constraints:

$$\sum_{i \in \mathcal{N}, f \in \mathcal{F}} P_{f,i,\varphi} = 0, \quad \sum_{i \in \mathcal{N}, f \in \mathcal{F}} Q_{f,i,\varphi} = 0 \quad \forall \varphi \neq \phi \quad (1p)$$

Results and challenges

P-Q flexibility areas for the 5-bus system:

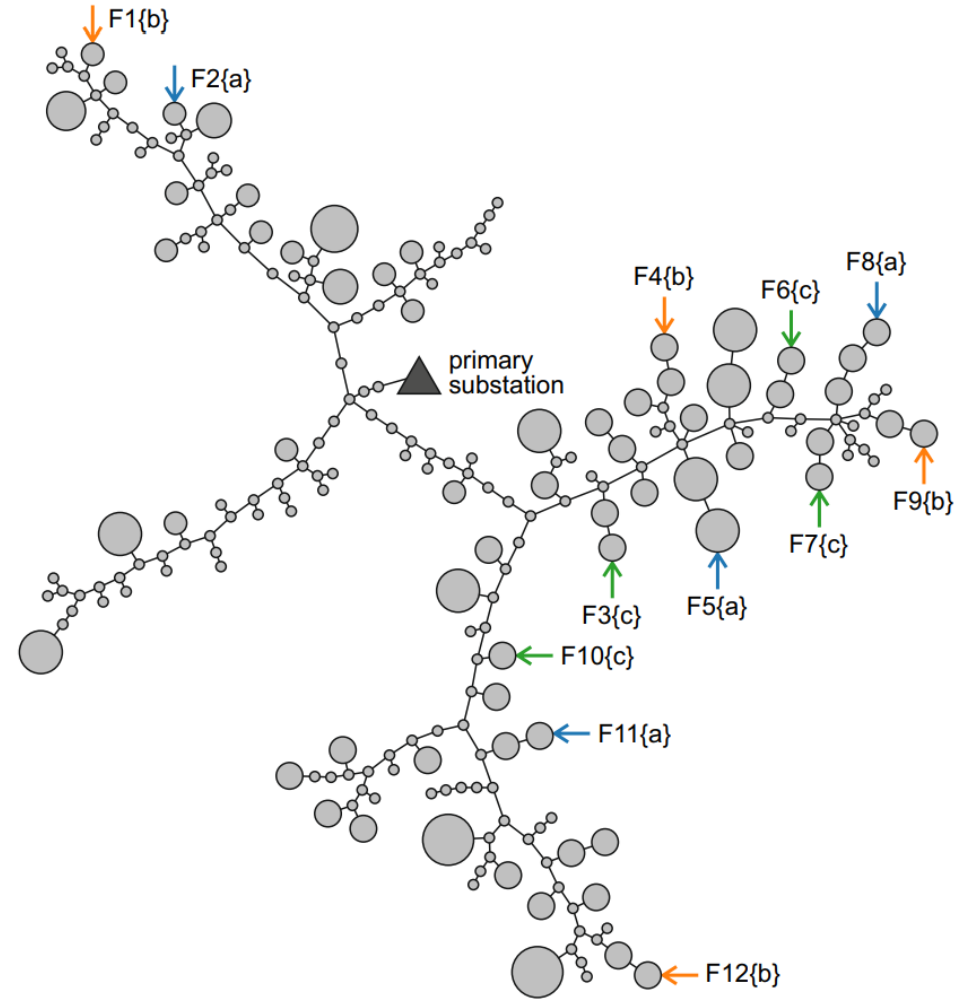


Challenge: Phase and voltage unbalance limits the aggregated P-Q flexibility

Challenge: Flexibility potential can be lost due to lack of coordination across phases

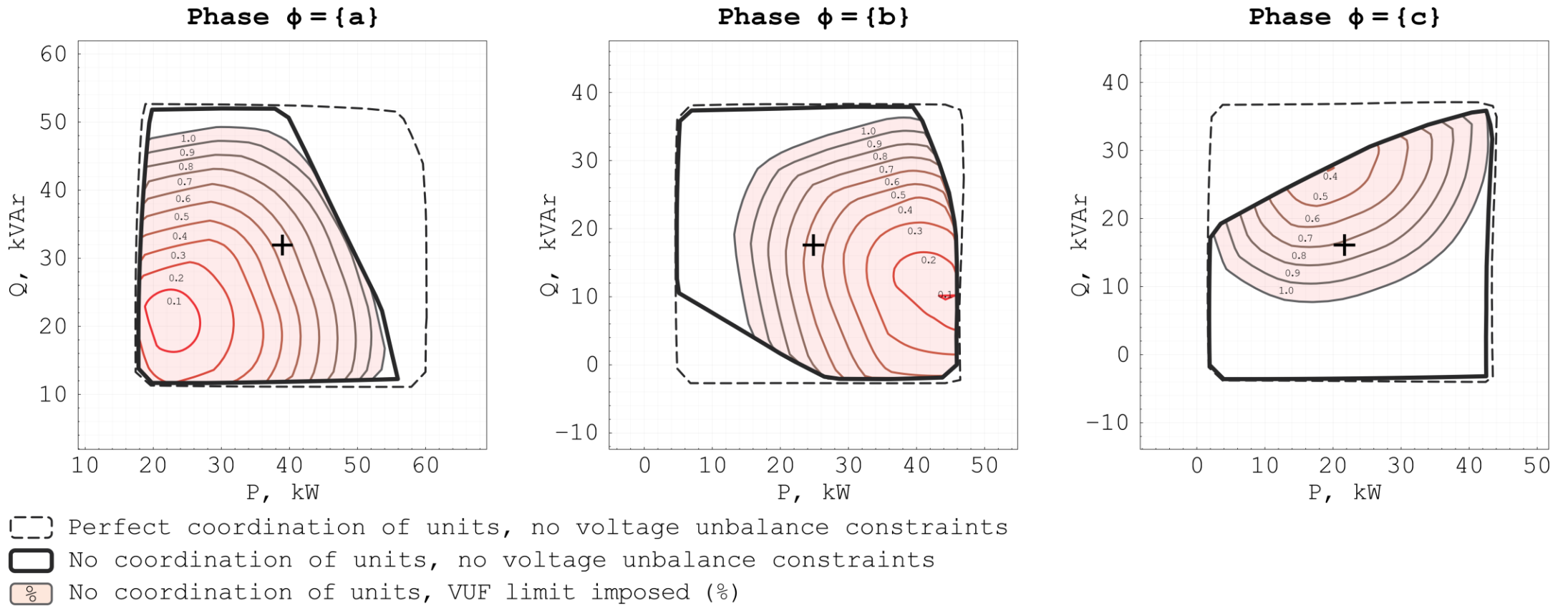
Results and challenges

Real 221-bus LV distribution network in the UK with 12 single-phase flexible units



Results and challenges

P-Q flexibility areas for the 221-bus system:



Challenge: Phase and voltage unbalance limits the aggregated P-Q flexibility

Challenge: Flexibility potential can be lost due to lack of coordination across phases

Conclusion

DER flexibility aggregation models are becoming important tools for distribution systems.

However, there are multiple **barriers and challenges** to overcome:

- Nonlinearity and nonconvexity
- Rapid changes in flexible unit dispatch
- Lack of DER coordination
- Impacts of phase unbalance

→ There is a need for improved accurate flexibility aggregation tools applicable to real distribution networks

Tracing, Ranking and Valuation of Aggregated DER Flexibility in Active Distribution Networks

Andrey Churkin, *Member, IEEE*, Wangwei Kong, *Member, IEEE*, Jose N. Melchor Gutierrez, Eduardo A. Martínez Ceseña, *Member, IEEE*, and Pierluigi Mancarella, *Senior Member, IEEE*

Abstract—The integration of distributed energy resources (DER) makes active distribution networks (ADNs) natural providers of flexibility services. However, the optimal operation of flexible units in ADNs is highly complex, which poses challenges for distribution system operators (DSOs) in aggregating DER flexibility. For example, to maximise the provision of services, flexible units must be strongly coordinated to manage network constraints, e.g., by performing simultaneous flexible power consumption and production in different parts of the network (referred to as power swaps in this work). Furthermore, due to the nonlinearities of aggregated DER flexibility provision, some units may need to rapidly change their outputs to enable the services. To address these challenges, this paper brings together exact AC optimal power flow (OPF) models and a cooperative game formulation and presents a new framework for tracing, ranking, and valuation of aggregated DER flexibility in ADNs. Extensive tests and simulations performed for the 33-bus radial distribution network demonstrate that the framework enables translating complex DER interactions into useful information for DSOs by ranking the criticality of flexible units and performing flexibility valuation based on its cost or economic surplus. Additionally, the framework proposes no-swap constraints and a nonlinearity metric which can be used by DSOs to identify unreliable operating regions with power swaps or rapid changes in flexible unit dispatch.

Index Terms—Active distribution network (ADN), Cooperative Game Theory, distributed energy resources (DER), flexibility services, nonlinearity, Shapley value, TSO-DSO coordination.

I. INTRODUCTION

THE increasing integration of distributed energy resources (DER) and flexible consumers makes active distribution networks (ADNs) natural providers of flexibility services [1], [2]. Such services can be utilised within distribution networks, as well as aggregated and traded between distribution system

as sets of feasible operating points in the P-Q space [8]–[15]. Such sets are known as flexibility P-Q areas, flexible power capability charts, or nodal operating envelopes. A thorough comparison of models and approximation methods for aggregated flexibility estimation is given in [16]–[18].

Regardless of their benefits, current studies on flexibility P-Q areas focus on estimating the limits (boundary) of the aggregated flexible power provision, overlooking the optimal flexible unit dispatch and contributions of individual units to aggregated flexibility.¹ Moreover, the P-Q areas are formulated under the potentially unrealistic assumption that all flexible units are perfectly coordinated and can perform fast flexible power control. However, the complexity and nonlinearity of the optimal flexible unit dispatch can pose significant challenges for DSOs. This work demonstrates two major challenges to the aggregated DER flexibility provision:

- 1) **Flexible power swaps** can occur when flexible units cannot follow purely economic incentives due to network constraints (e.g., voltage and thermal limits). In such cases, some units have to manage network constraints to enable other units to provide flexibility services (and potentially earn most of the revenue from the provision of services). For example, some units may need to produce flexible power (to manage voltage and thermal constraints) whereas other units consume it. DSOs can provide aggregated flexibility in such regimes only under the assumption of perfect unit coordination. However, this assumption may be unrealistic, as some units may only be partially controllable by DSOs, and not all units may exchange information. Recent studies, such as [19], highlighted the need for developing new mod-

Exposing Barriers to Flexibility Aggregation in Unbalanced Distribution Networks

Andrey Churkin[♦], *Member, IEEE*, Wangwei Kong[♦], *Member, IEEE*, Pierluigi Mancarella, *Fellow, IEEE*, and Eduardo A. Martínez Ceseña, *Member, IEEE*

Abstract—The increasing integration of distributed energy resources (DER) offers new opportunities for distribution system operators (DSO) to improve network operation through flexibility services. To utilise flexible resources, various DER flexibility aggregation methods have been proposed, such as the concept of aggregated P-Q flexibility areas. Yet, many existing studies assume perfect coordination among DER and rely on single-phase power flow analysis, thus overlooking barriers to flexibility aggregation in real unbalanced systems. To quantify the impact of these barriers, this paper proposes a three-phase optimal power flow (OPF) framework for P-Q flexibility assessment, implemented as an open-source Julia tool `3FlexAnalyser.jl`. The framework explicitly accounts for voltage unbalance and imperfect coordination among DER in low voltage (LV) distribution networks. Simulations on an illustrative 5-bus system and a real 221-bus LV network in the UK reveal that over 30% of the theoretical aggregated flexibility potential can be lost due to phase unbalance and lack of coordination across phases. These findings highlight the need for improved flexibility aggregation tools applicable to real unbalanced distribution networks.

Index Terms—Aggregated flexibility, distribution network, distributed energy resources (DER), flexibility services, low voltage networks, phase unbalance, voltage unbalance.

I. INTRODUCTION

MODERN distribution systems are evolving rapidly with the increasing integration of distributed energy resources (DER), particularly in low voltage (LV) networks [1]–[3]. Controllable DER, such as battery energy storage systems (BESS), prosumers, and electric vehicles (EV), have the technical ability to adjust their power exchange with the

[3], [5], [12]–[31]. These models aim to estimate the limits of flexible active (P) and reactive (Q) power exchanges at specific locations of a distribution network. By estimating the P-Q limits, DSO can clear flexibility markets while managing network constraints. The sets of all feasible power exchanges (operating points) in the P-Q space are known as aggregated P-Q flexibility areas or nodal operating envelopes [5], [26].

Despite recent advances in DER flexibility aggregation models, several critical assumptions regarding DER operation in unbalanced LV networks remain largely unchallenged, making it easy to overlook barriers to flexibility aggregation in real-world distribution networks. Specifically, this paper exposes two barriers that can significantly reduce the usable flexibility from DER:

- **Barrier #1:** Lack of DER coordination. Most flexibility aggregation models assume that all flexibility providers (referred to as flexible units in this work) can be perfectly coordinated [5], [10], [12]–[20], [22]–[25], [27]–[31]. Such centralised aggregation methods imply that units at different locations can jointly adjust their power in order to manage voltage constraints and keep the network's operation feasible. However, in real distribution systems, flexible units connected to different phases across the network may not be perfectly coordinated due to low observability, data exchange issues, single-phase connection, behind-the-meter devices, and other constraints [21], [26]. These practical limitations of DER coordination create a barrier to effective flexibility aggregation.



<https://github.com/AndreyChurkin/3FlexAnalyser.jl>

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Resources

More materials on my YouTube channel <https://www.youtube.com/@ChuScience>



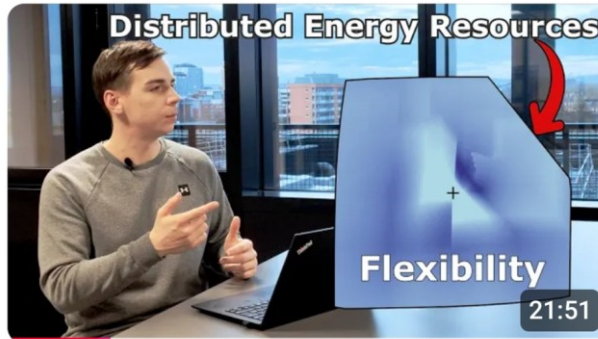
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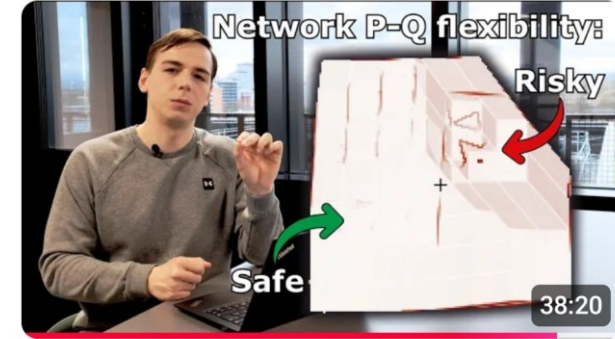
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Thank you!

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