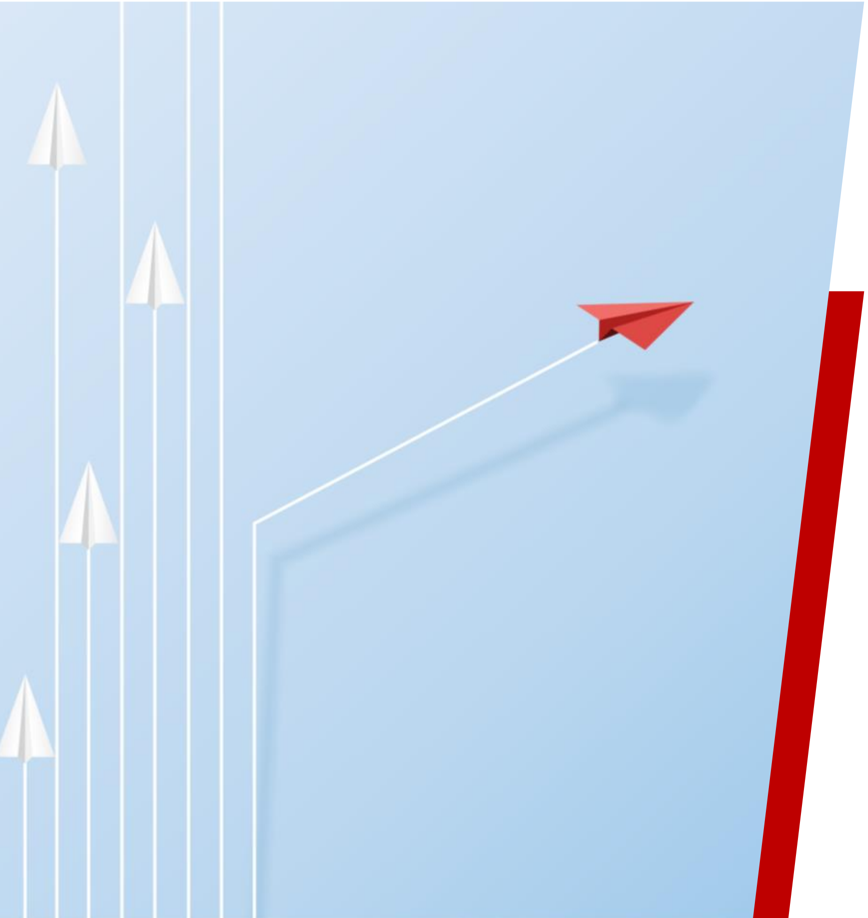


Exploring feedback Nash equilibria in infinte-horizon LQ dynamic games

Game On! Seminar

24th February 2026 – Benita Nortmann

benita.nortmann@empa.ch

Motivation

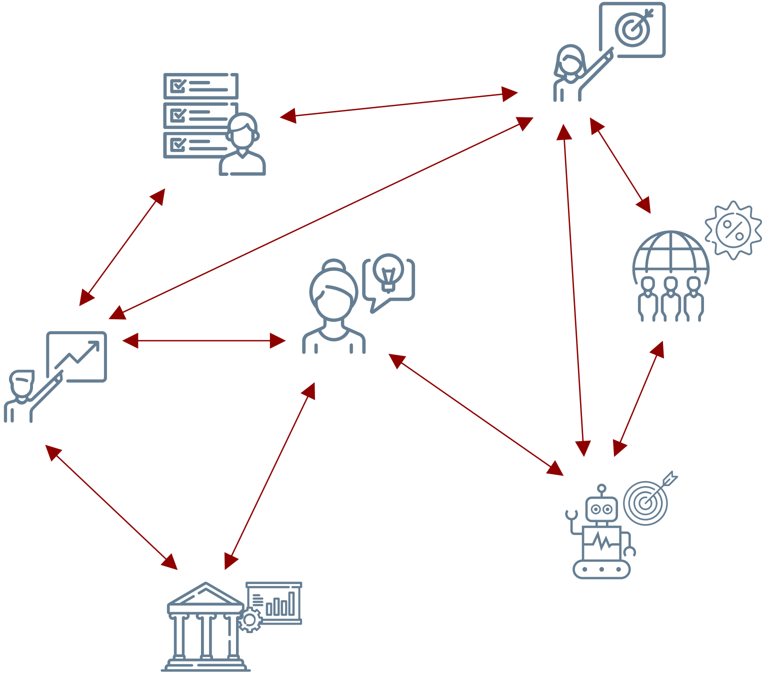
Feedback Nash equilibria

Iterative methods

Data-driven methods

Motivation

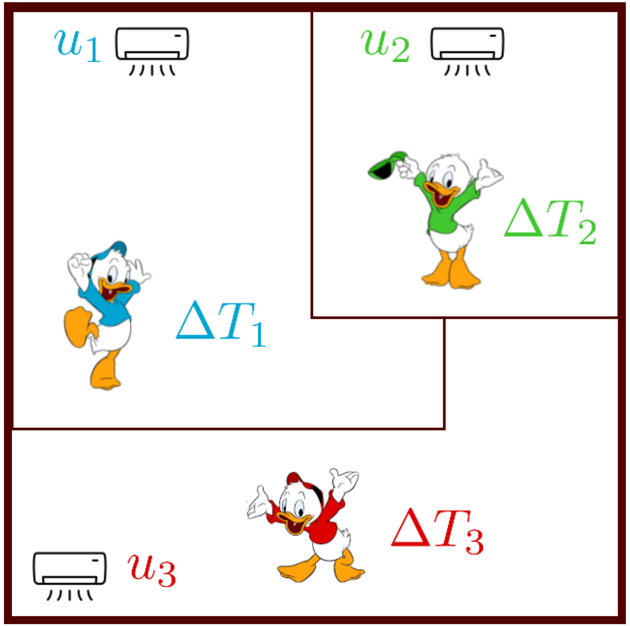
Dynamic multi-player decisions



Example: building thermal control

Motivation: NEST

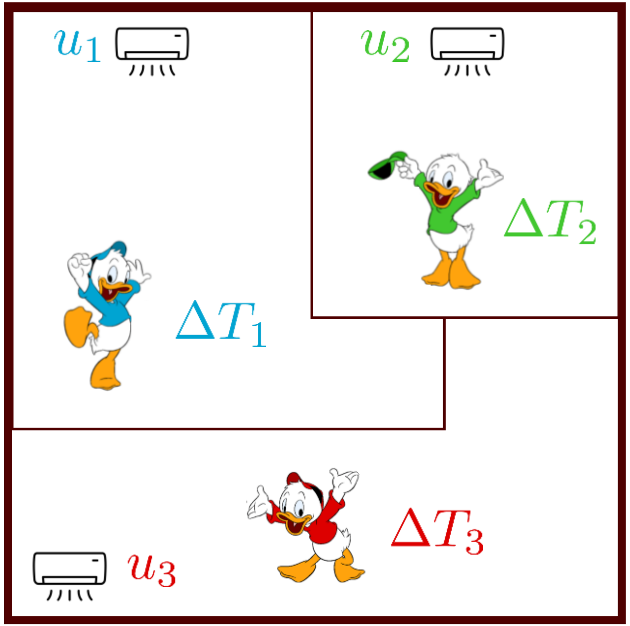
Example: building thermal control



Motivation: NEST

$$\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}(k+1) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}(k) + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u_1(k) + \begin{bmatrix} 0 \\ b_2 \\ 0 \end{bmatrix} u_2(k) + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} u_3(k) + d(k)$$

Example: building thermal control



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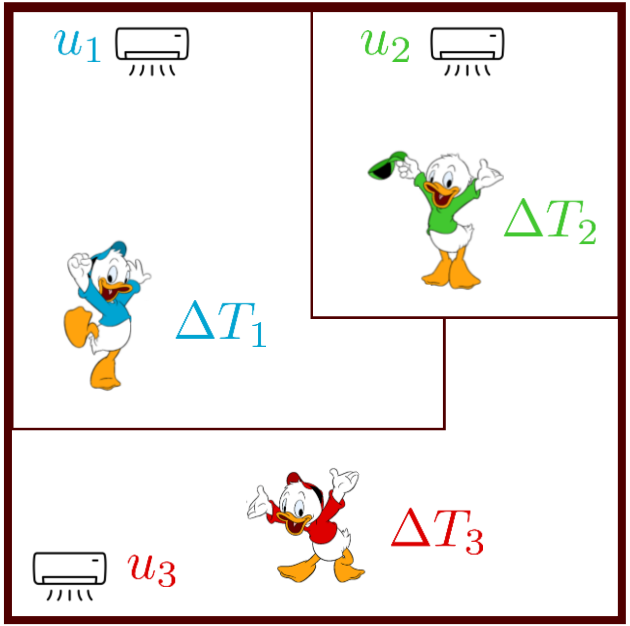
Example: building thermal control



Motivation: NEST

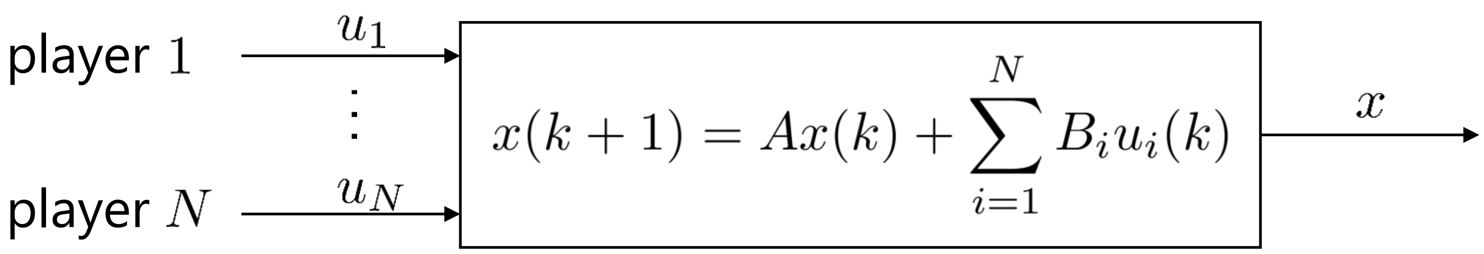
$$J_1 = \sum_{k=0}^{\infty} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}^\top \begin{bmatrix} q_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} + r_1 u_1^2$$

$$J_2 = \sum_{k=0}^{\infty} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}^\top \begin{bmatrix} 0 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} + r_2 u_2^2$$

$$J_3 = \sum_{k=0}^{\infty} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}^\top \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} + r_3 u_3^2$$

$$\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}(k+1) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}(k) + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u_1(k) + \begin{bmatrix} 0 \\ b_2 \\ 0 \end{bmatrix} u_2(k) + \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} u_3(k)$$

Linear-quadratic dynamic game



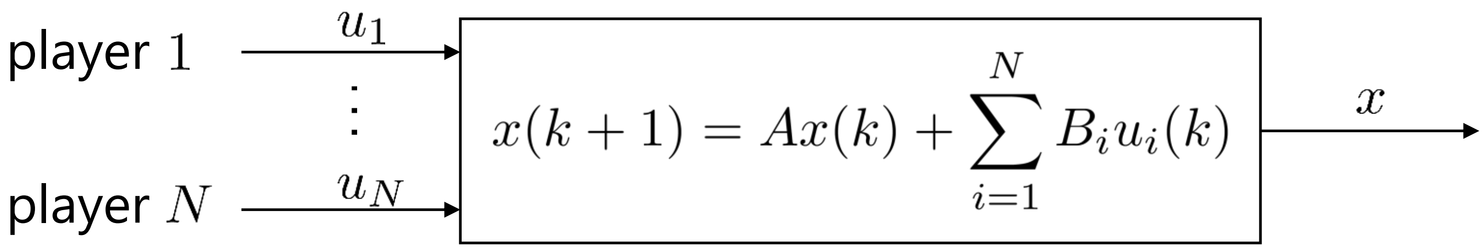
Each player i aims to minimise

$$J_i(x(0), u_i(\cdot), u_{-i}(\cdot)) = \sum_{k=0}^{\infty} \left(x(k)^\top Q_i x(k) + u_i(k)^\top R_i u_i(k) \right)$$

with $Q_i = Q_i^\top \succeq 0$, $R_i = R_i^\top \succ 0$, for $i = 1, \ldots, N$, and

$$u_{-i} = \{u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_N\}$$

Linear-quadratic dynamic game



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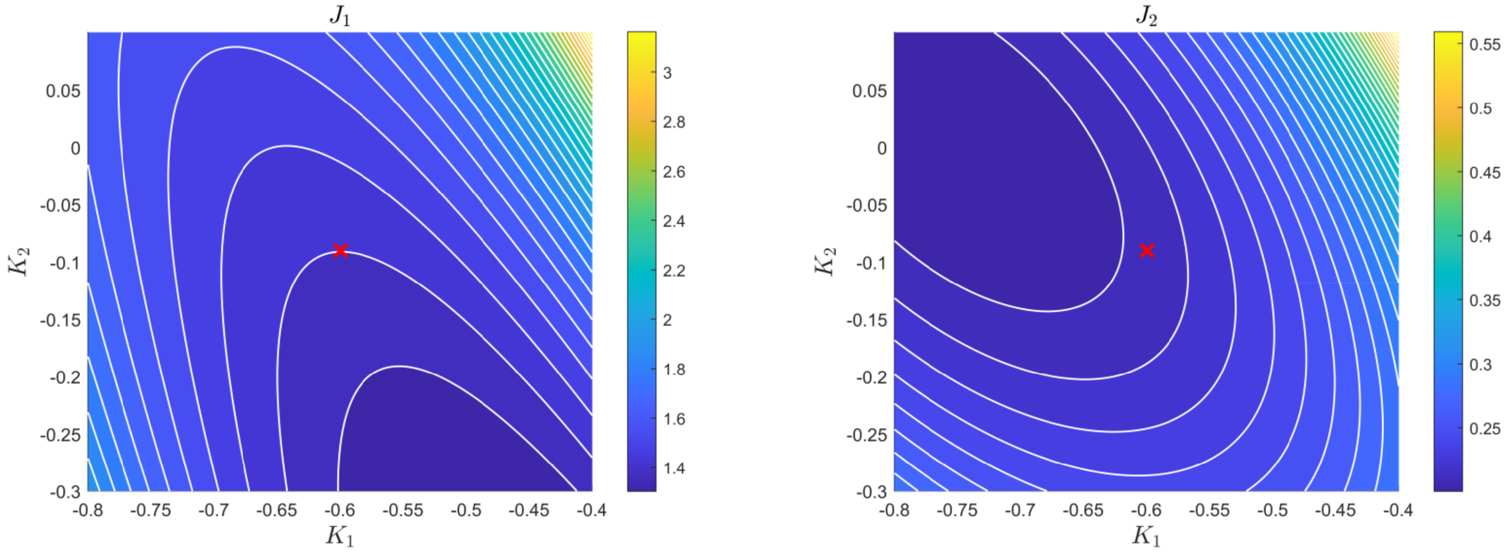
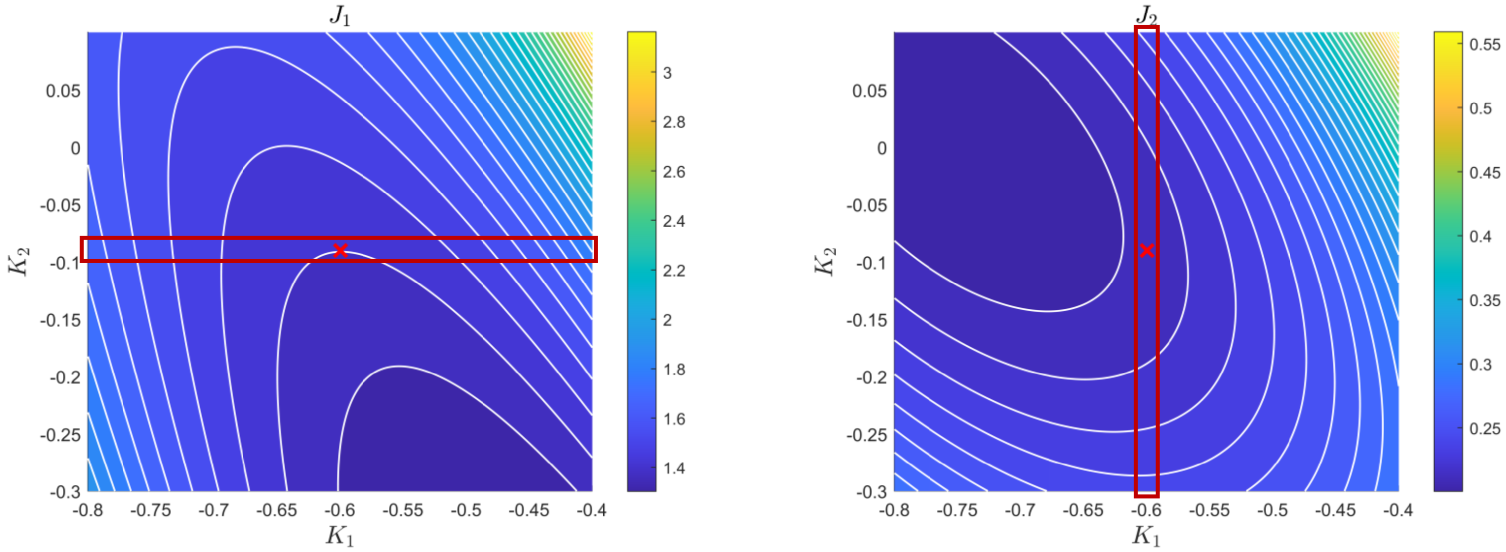
Feedback Nash equilibria

Nash equilibrium solutions



$$J_i^\star = J_i(x(0), u_i^\star(\cdot), u_{-i}^\star(\cdot)) \leq J_i(x(0), u_i(\cdot), u_{-i}^\star(\cdot))$$

Nash equilibrium solutions



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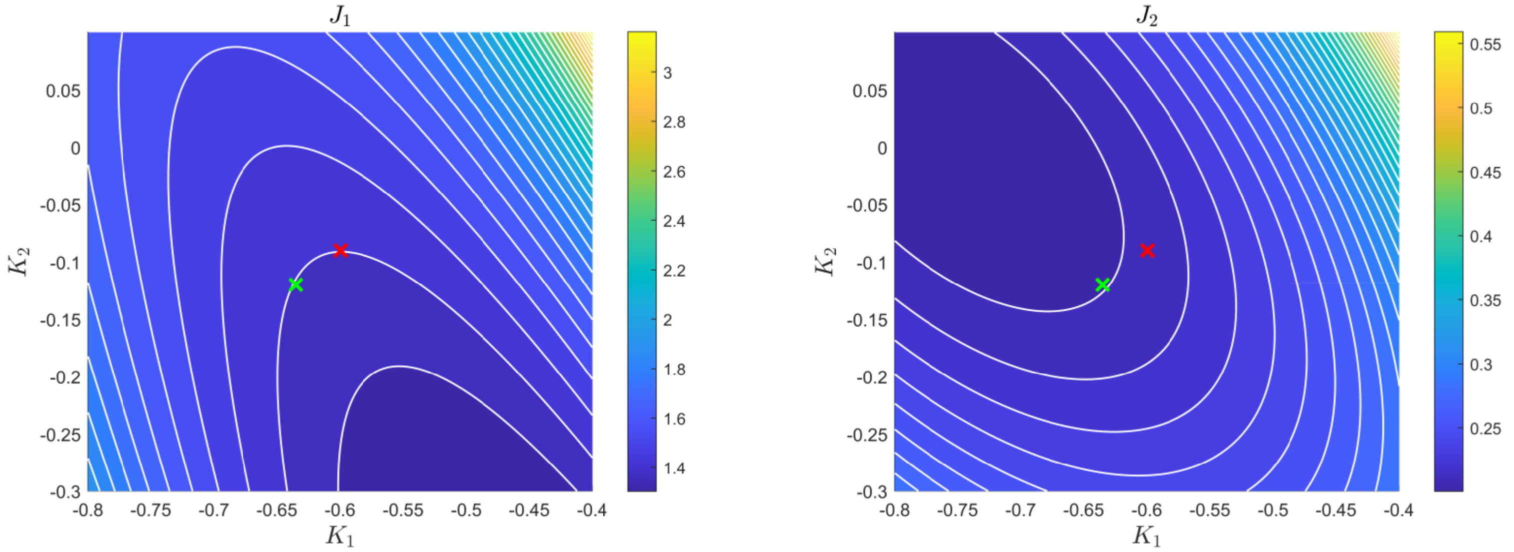

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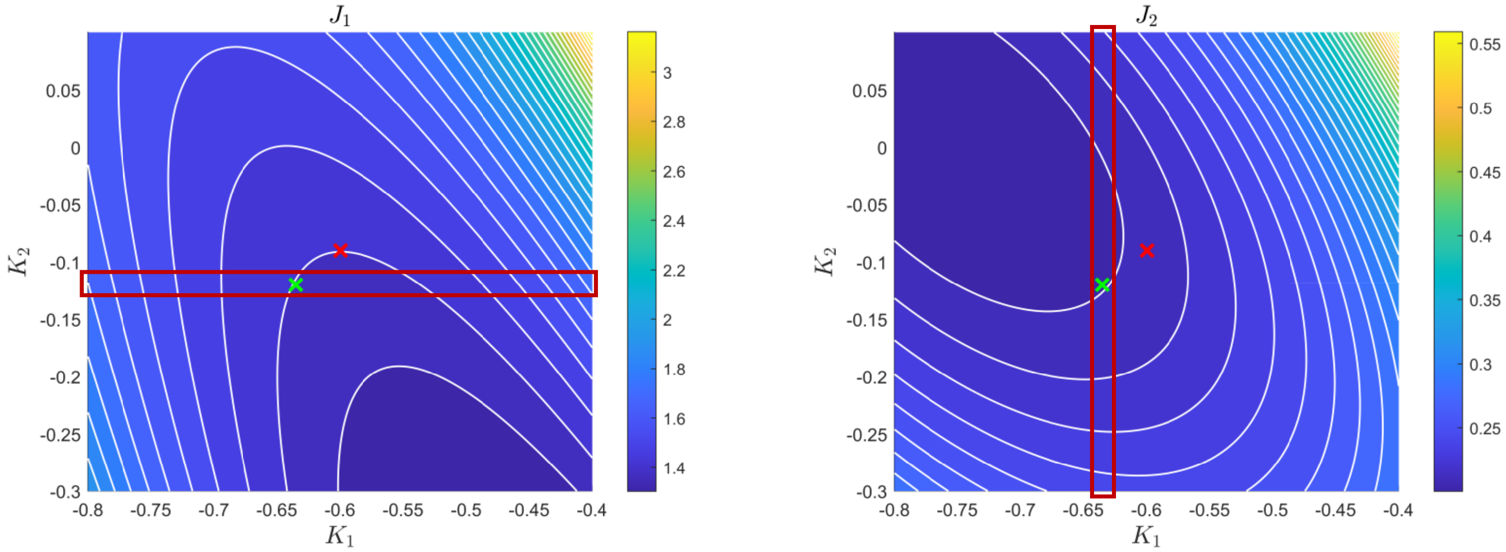

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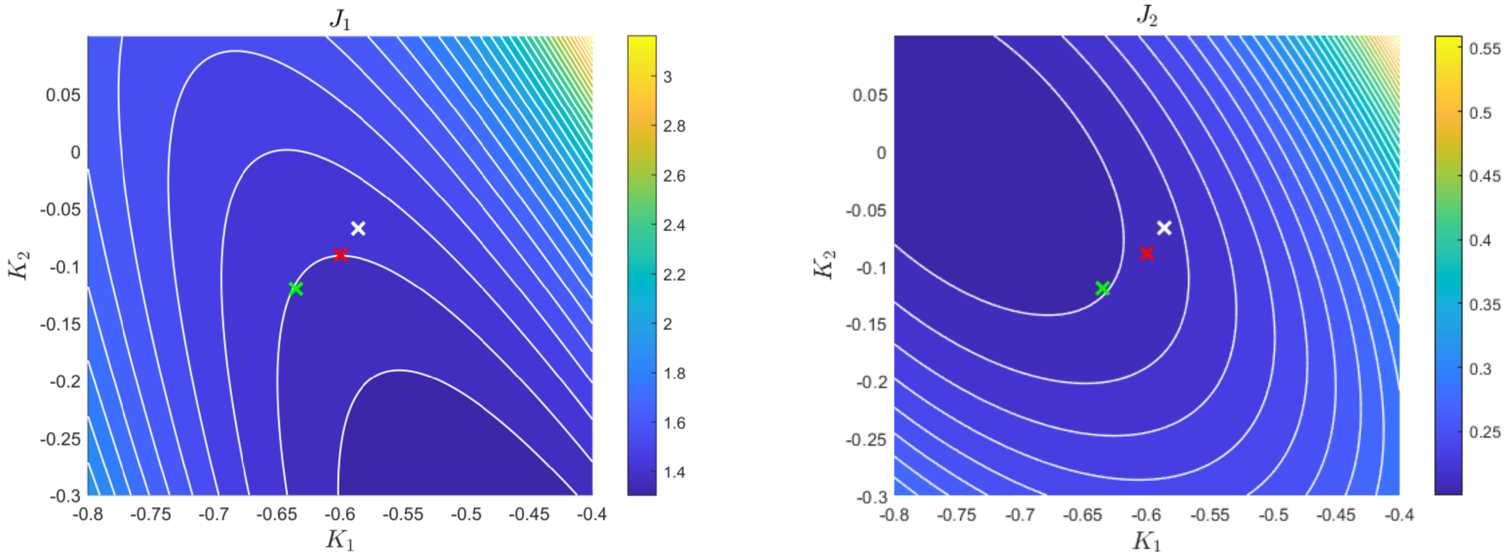

$$J_i^\star = J_i(x(0), u_i^\star(\cdot), u_{-i}^\star(\cdot)) \leq J_i(x(0), u_i(\cdot), u_{-i}^\star(\cdot))$$

Nash equilibrium solutions

$$J_i(x_0, \phi_i^\star(\cdot), \phi_{-i}^\star(\cdot)) \leq J_i(x_0, \phi_i(\cdot), \phi_{-i}^\star(\cdot))$$

Open-loop

$$u_i^\star(k) = \phi_i(x_0, k)$$

Variational methods

Feedback

$$u_i^\star(k) = \phi_i(x(k))$$

Dynamic programming

Nash equilibrium solutions

$$x(k+1) = Ax(k) + B_i u_i(k) + \sum_{j \neq i}^{N} B_j u_j(k)$$

$$J_i(x_0, \phi_i^\star(\cdot), \phi_{-i}^\star(\cdot)) \leq J_i(x_0, \phi_i(\cdot), \phi_{-i}^\star(\cdot))$$

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$$u_i^\star(k) = \phi_i(x_0, k)$$

sometimes: $= \phi_i(x^\star(k))$

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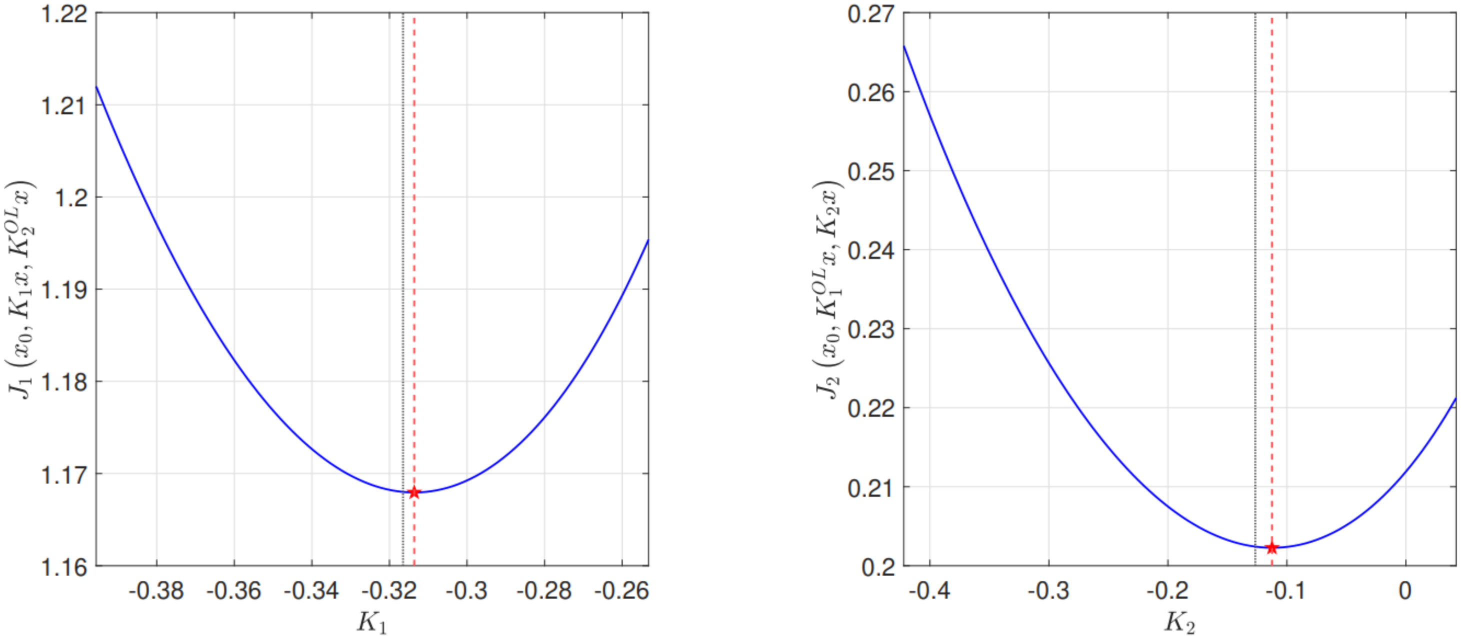
Open-loop		Feedback
$u_i^\star(k) = \phi_i(x_0, k)$	$\neq$	$u_i^\star(k) = \phi_i(x(k))$
sometimes: $= \phi_i(x^\star(k))$		
Variational methods		Dynamic programming
Weakly time consistent		Strongly time consistent

Nash equilibrium solutions



$$J_1(x_0, K_1^{OL}\hat{x}(\cdot), K_2^{OL}\hat{x}(\cdot)) \geq J_1(x_0, \hat{K}_1\hat{x}(\cdot), K_2^{OL}\hat{x}(\cdot))$$

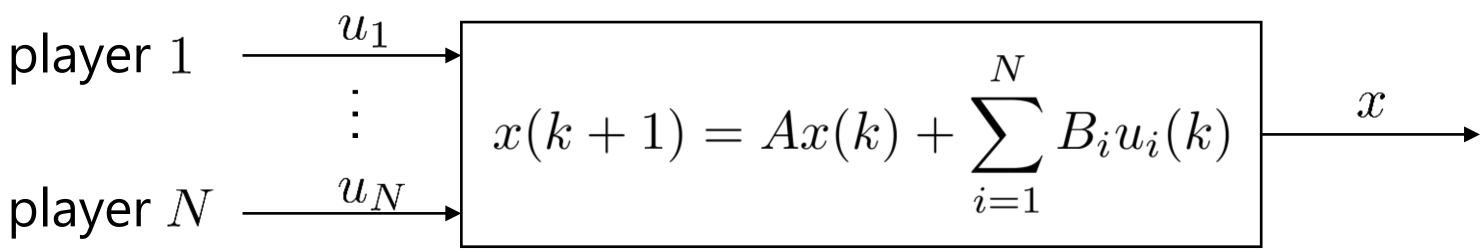
A. Monti, B. Nortmann, T. Mylvaganam, and M. Sassano, “Feedback and open-loop Nash equilibria for LQ infinite-horizon discrete-time dynamic games,” SIAM Journal on Control and Optimization, vol. 62, no. 3, pp. 1417–1436, 2024.

Nash equilibrium solutions

$$J_i(x_0, \phi_i^\star(\cdot), \phi_{-i}^\star(\cdot)) \leq J_i(x_0, \phi_i(\cdot), \phi_{-i}^\star(\cdot))$$

Open-loop		Feedback
$u_i^\star(k) = \phi_i(x_0, k)$	$\neq$	$u_i^\star(k) = \phi_i(x(k))$
sometimes: $= \phi_i(x^\star(k))$		
Variational methods		Dynamic programming
Weakly time consistent		Strongly time consistent
Easier to compute		Harder to compute
Number & existence characterised by eigenstructure of Hamiltonian		Number & existence harder to check

Feedback Nash equilibrium solutions



Each player i aims to minimise

$$J_i(x(0), u_i(\cdot), u_{-i}(\cdot)) = \sum_{k=0}^{\infty} \Big(x(k)^\top Q_i x(k) + u_i(k)^\top R_i u_i(k) \Big)$$

Find $u_i^\star(k) = K_i^\star x(k)$ such that

$$J_i^\star = J_i(x(0), u_i^\star(\cdot), u_{-i}^\star(\cdot)) \leq J_i(x(0), u_i(\cdot), u_{-i}^\star(\cdot))$$

for all stabilising $\{u_i, u_{-i}^\star\}$, $i = 1, \ldots, N$, $i \neq -i$.

Feedback Nash equilibrium solutions

Characterised via coupled algebraic matrix equations

$$0 = Q_i + {K_i^\star}^\top R_i K_i^\star + \left(A + \sum_{j=1}^{N} B_j K_j^\star\right)^\top P_i^\star \left(A + \sum_{j=1}^{N} B_j K_j^\star\right) - P_i^\star$$

$$0 = \left(R_i + B_i^\top P_i^\star B_i\right) K_i^\star + B_i^\top P_i^\star \left(A + \sum_{j=1, j\neq i}^{N} B_j K_j^\star\right)$$

for $i = 1, \ldots, N$, with $P_i^\star = {P_i^\star}^\top \succ 0$, and

$$u_i^\star(k) = K_i^\star x(k) \qquad J_i^\star = x(k)^\top P_i^\star x(k)$$

Feedback Nash equilibrium solutions

Characterised via coupled algebraic matrix equations

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Feedback Nash equilibrium solutions

Eliminating $P_i^\star$ and rearranging gives $n \sum_{i=1}^{N} m_i$ polynomial equations

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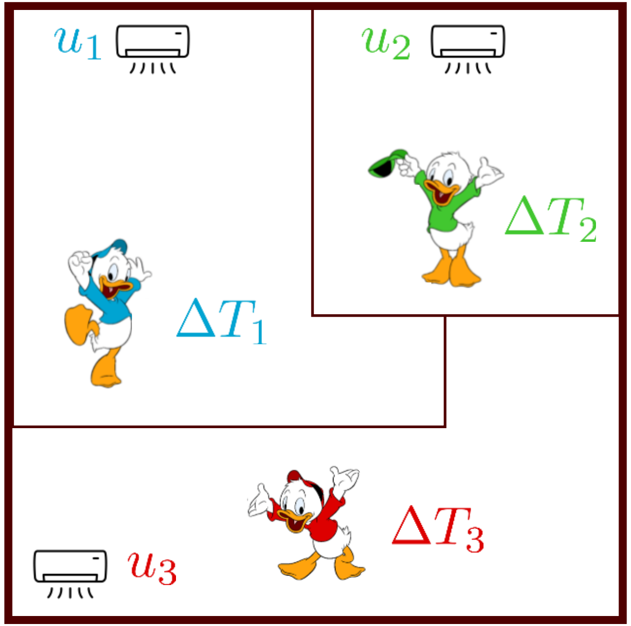

number of equations: 9

polynomial degree: 12

How many solutions are there?

Let's build intuition by focusing on the *scalar case*

Dynamics: $$x(k+1) = ax(k) + \sum_{i=1}^{N} b_i u_i(k)$$

Player i aims to minimise: $$J_i(x_0, u_1, \ldots, u_N) = \sum_{k=0}^{\infty} \left(q_i x(k)^2 + r_i u_i(k)^2\right)$$

Coupled equations characterising FNE solutions:

$$r_i k_i^\star \left(a_{cl}^2 - 1\right) - b_i a_{cl} \left(r_i {k_i^\star}^2 + q_i\right) = 0$$

For $i = 1, \ldots, N$, where $a_{cl} := a + \sum_{j=1}^{N} b_j k_j^\star$

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For $i = 1, \ldots, N$, where $a_{cl} := a + \sum_{j=1}^{N} b_j k_j^\star$

How many solutions are there?

Alternative characterisation of FNE solutions:

$$a = \hat{f}(\xi) + N\xi + t_1\sqrt{\xi^2 - \sigma_1} + \ldots + t_N\sqrt{\xi^2 - \sigma_N}$$

where $$t_i \in \{-1, 1\}, \quad \sigma_i := \frac{b_i^2 q_i}{r_i}$$

$$\hat{f}(\xi) := \begin{cases} -\xi - \sqrt{\xi^2 + 1} & \text{if } \xi < 0 \\ -\xi + \sqrt{\xi^2 + 1} & \text{if } \xi > 0 \end{cases}$$

2^N univariate "auxiliary" functions in ξ

and FNE strategies of player i are given by

$$u_i^\star(k) = k_i^\star x(k) = \frac{-\xi - t_i\sqrt{\xi^2 - \sigma_i}}{b_i} x(k)$$

B. Nortmann, M. Sassano, and T. Mylvaganam, "Feedback Nash equilibria for scalar N-player linear quadratic dynamic games", Automatica, vol. 174, p. 112133, 2025.

Continuous-time counterpart: J. Engwerda, "Properties of feedback Nash equilibria in scalar LQ differential games", Automatica, vol. 69, p. 364–374, 2016.

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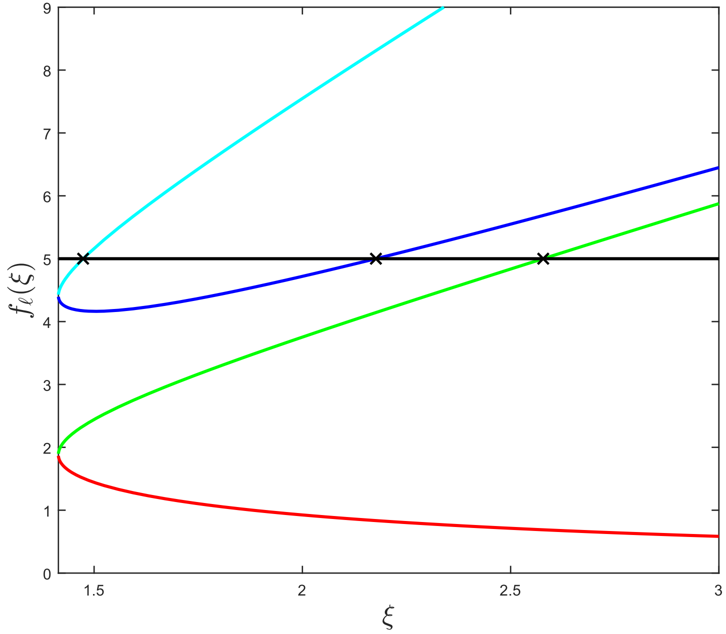
Alternative characterisation of FNE solutions:

$$a = \hat{f}(\xi) + N\xi + t_1\sqrt{\xi^2 - \sigma_1} + \ldots + t_N\sqrt{\xi^2 - \sigma_N}$$

For specific problems:

Visualise number and certain properties of FNE solutions *without solving* coupled equations



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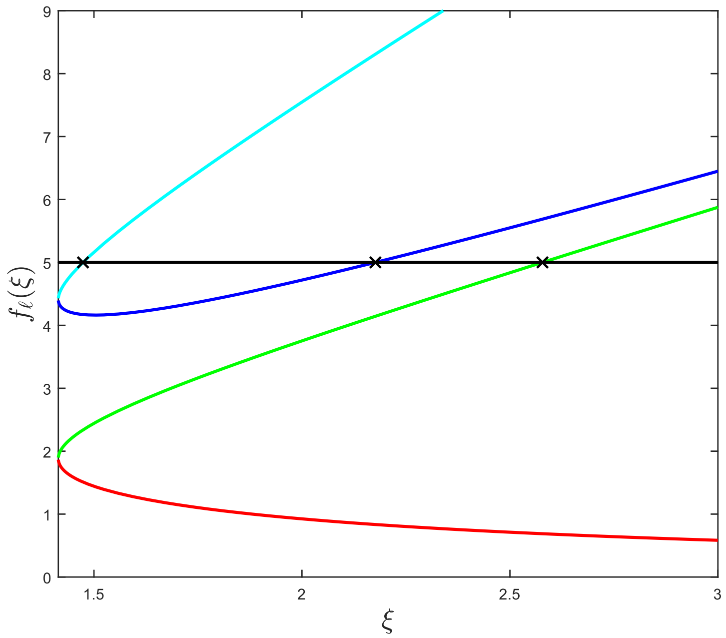
Alternative characterisation of FNE solutions:

$$a = \hat{f}(\xi) + N\xi + t_1\sqrt{\xi^2 - \sigma_1} + \ldots + t_N\sqrt{\xi^2 - \sigma_N}$$

For generic problem parameters:

Allows us to derive *bounds on number* of solutions, e.g.

- max nr. of FNE solutions: $2^N - 1$
- if $|a| < 1$ there exists a unique FNE



B. Nortmann, M. Sassano, and T. Mylvaganam, "Feedback Nash equilibria for scalar N-player linear quadratic dynamic games", Automatica, vol. 174, p. 112133, 2025.

Iterative methods

Finding feedback Nash equilibrium solutions

Goal: iteratively determine a stabilising solution of the coupled algebraic matrix equations

$$0 = Q_i + {K_i^\star}^\top R_i K_i^\star + \left(A + \sum_{j=1}^{N} B_j K_j^\star\right)^\top P_i^\star \left(A + \sum_{j=1}^{N} B_j K_j^\star\right) - P_i^\star$$

$$0 = \left(R_i + B_i^\top P_i^\star B_i\right) K_i^\star + B_i^\top P_i^\star \left(A + \sum_{j=1, j\neq i}^{N} B_j K_j^\star\right)$$

for $i = 1, \dots, N$, with $P_i^\star = {P_i^\star}^\top \succ 0$, and

$$u_i^\star(k) = K_i^\star x(k) \qquad J_i^\star = x(k)^\top P_i^\star x(k)$$

For continuous-time case see e.g.:

T.-Y. Li and Z. Gajic, "Lyapunov Iterations for Solving Coupled Algebraic Riccati Equations of Nash DifferentialGames and Algebraic Riccati Equations of Zero-Sum Games," in New Trends in Dynamic Games and Applications,Birkhäuser Boston, pp. 333–351, 1995.

J. Engwerda, "Algorithms for computing Nash equilibria in deterministic LQ games," Computational Management Science, vol. 4, no. 2, pp. 113–140, 2007.

Iterative feedback Nash equilibrium finding

Updates involve solution of uncoupled equations for each player

Lyapunov updates:

$$0 = Q_i + K_i^{(l)\top} R_i K_i^{(l)} - P_i^{(l+1)} + \left(\hat{A}_{\sigma,i}^{(l+1)} + B_i K_i^{(l)}\right)^\top P_i^{(l+1)} \left(\hat{A}_{\sigma,i}^{(l+1)} + B_i K_i^{(l)}\right)$$

$$K_i^{(l+1)} = -\left(R_i + B_i^\top P_i^{(l+1)} B_i\right)^{-1} B_i^\top P_i^{(l+1)} \hat{A}_{\sigma,i}^{(l+1)}$$

Riccati updates:

$$0 = Q_i + K_i^{(l+1)\top} R_i K_i^{(l+1)} - P_i^{(l+1)} + \left(\hat{A}_{\sigma,i}^{(l+1)} + B_i K_i^{(l+1)}\right)^\top P_i^{(l+1)} \left(\hat{A}_{\sigma,i}^{(l+1)} + B_i K_i^{(l+1)}\right)$$

$$K_i^{(l+1)} = -\left(R_{ii} + B_i^\top P_i^{(l+1)} B_i\right)^{-1} B_i^\top P_i^{(l+1)} \hat{A}_{\sigma,i}^{(l+1)}$$

B. Nortmann, A. Monti, M. Sassano and T. Mylvaganam, "Nash Equilibria for Linear Quadratic Discrete-Time Dynamic Games via Iterative and Data-Driven Algorithms", IEEE Transactions on Automatic Control, vol. 69, no. 10, pp. 6561-6575, 2024.

Iterative feedback Nash equilibrium finding

Synchronous

$$\left\{K_1^{(l)}, \ldots, K_N^{(l)}\right\}$$

All players update simultaneously

$$\left\{K_1^{(l+1)}, \ldots, K_N^{(l+1)}\right\}$$

$$\hat{A}_{\sigma,i}^{(l)} = A + \sum_{j=1, j\neq i}^{N} B_j K_j^{(l-1)}$$

Asynchronous

$$\left\{K_1^{(l)}, \ldots, K_N^{(l)}\right\}$$

$$\vdots$$

Player i update

$$\left\{K_1^{(l+1)}, \ldots, K_i^{(l+1)}, K_{i+1}^{(l)}, \ldots, K_N^{(l)}\right\}$$

Player $i+1$ update

$$\vdots$$

$$\left\{K_1^{(l+1)}, \ldots, K_N^{(l+1)}\right\}$$

$$\hat{A}_{\sigma,i}^{(l)} = A + \sum_{w=1}^{i-1} B_w K_w^{(l)} + \sum_{j=i+1}^{N} B_j K_j^{(l-1)}$$

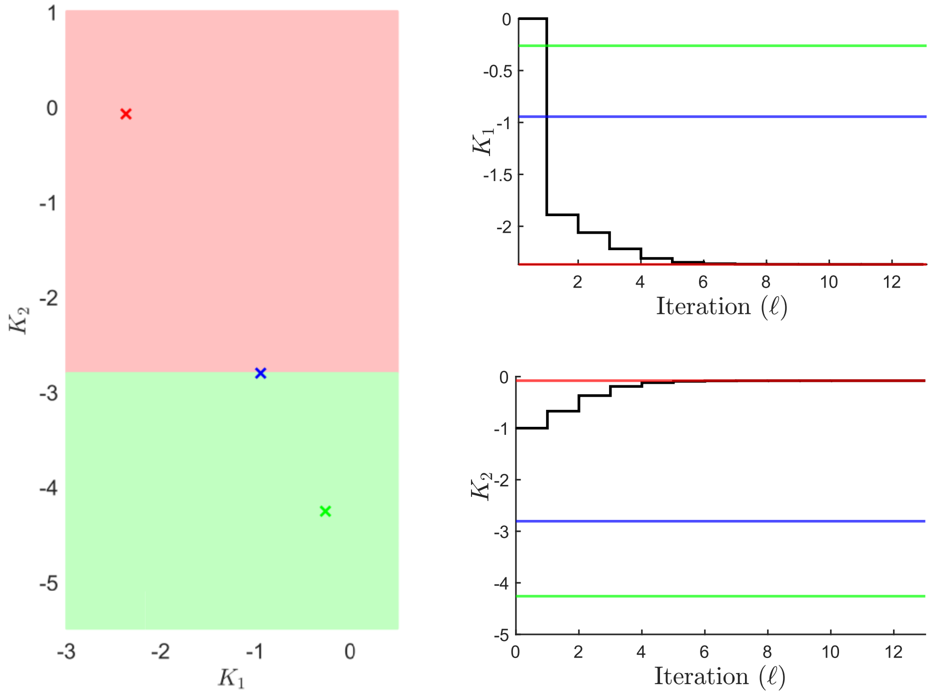
B. Nortmann, A. Monti, M. Sassano and T. Mylvaganam, “Nash Equilibria for Linear Quadratic Discrete-Time Dynamic Games via Iterative and Data-Driven Algorithms”, IEEE Transactions on Automatic Control, vol. 69, no. 10, pp. 6561-6575, 2024.

Iterative feedback Nash equilibrium finding

Discussing conditions for:

Local convergence

Recursive feasibility



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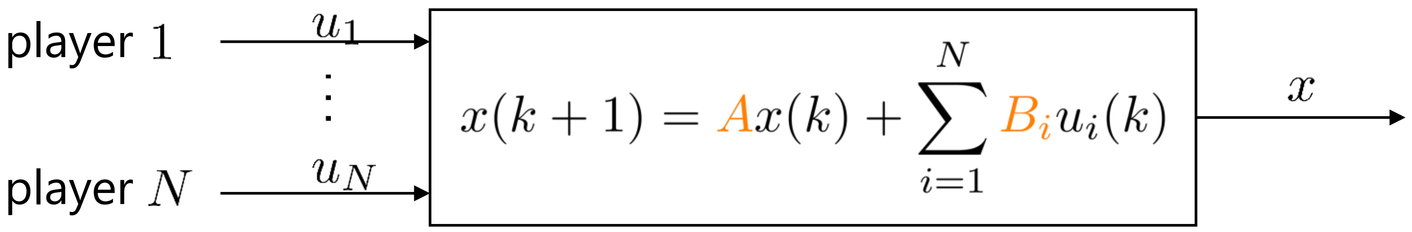
Data-driven methods

LQ game with incomplete information



$$J_1(x(0), u_1(\cdot), u_{-1}(\cdot)) = \sum_{k=0}^{\infty} \left(x(k)^\top Q_1 x(k) + u_1(k)^\top R_1 u_1(k) \right)$$

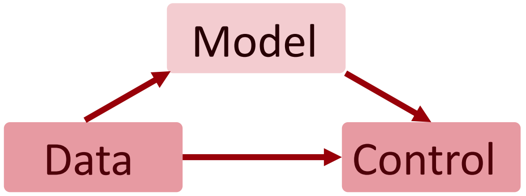
$$\vdots$$

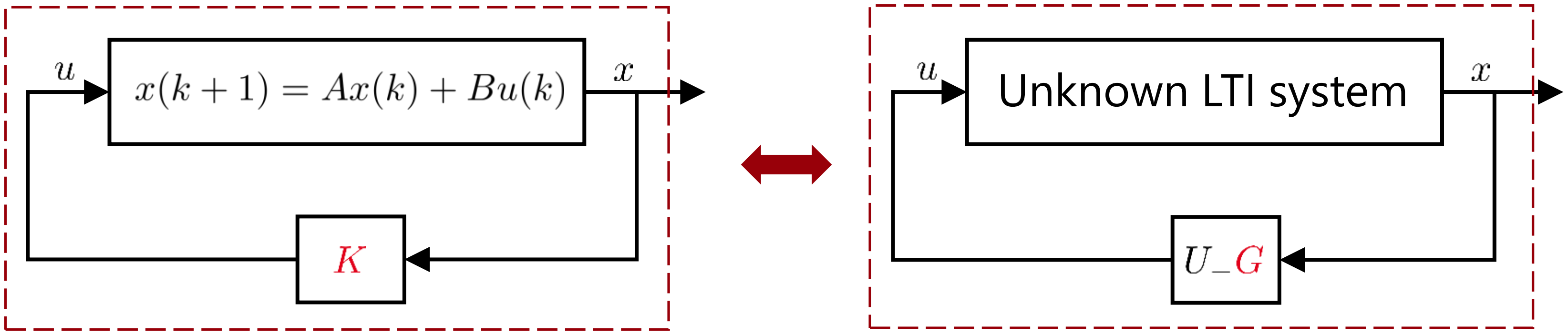
$$J_i(x(0), u_i(\cdot), u_{-i}(\cdot)) = \sum_{k=0}^{\infty} \left(x(k)^\top Q_i x(k) + u_i(k)^\top R_i u_i(k) \right)$$

$$\vdots$$

$$J_N(x(0), u_N(\cdot), u_{-N}(\cdot)) = \sum_{k=0}^{\infty} \left(x(k)^\top Q_N x(k) + u_N(k)^\top R_N u_N(k) \right)$$

Direct data-driven control





$x(k+1) = (A + BK)x(k)$

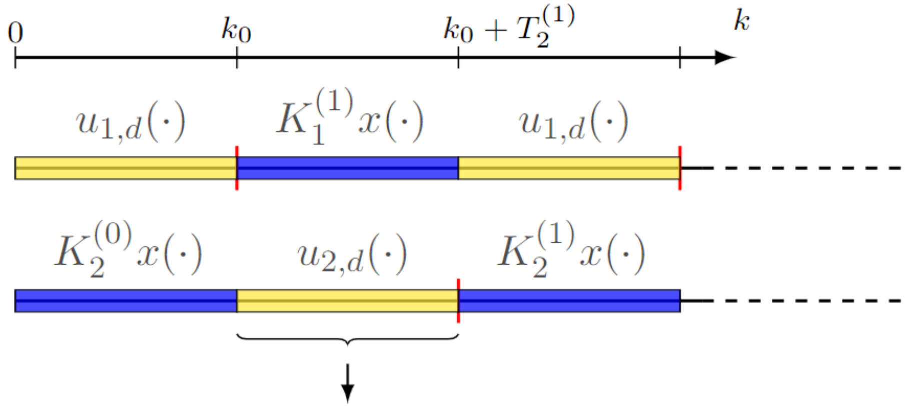
$x(k+1) = X_+ G x(k)$

$$
\begin{aligned}
X_- &= \begin{bmatrix} x(0) & \dots & x(T-1) \end{bmatrix} \in \mathbb{R}^{n \times T} \\
U_- &= \begin{bmatrix} u(0) & \dots & u(T-1) \end{bmatrix} \in \mathbb{R}^{m \times T} \\
X_+ &= \begin{bmatrix} x(1) & \dots & x(T) \end{bmatrix} \in \mathbb{R}^{n \times T}
\end{aligned}
$$

C. De Persis and P. Tesi, “Formulas for data-driven control: Stabilization, optimality, and robustness,” IEEE Transactions on Automatic Control, vol. 65, no. 3, pp. 909–924, 2020.

Data-driven iterative Nash equilibrium finding

Scheduling experiments and collecting data



$$U_{2-} = \begin{bmatrix} u_{2,d}(k_0) & \dots & u_{2,d}(k_0 + T_2^{(1)} - 1) \end{bmatrix}$$

$$X_{-} = \begin{bmatrix} x_d(k_0) & \dots & x_d(k_0 + T_2^{(1)} - 1) \end{bmatrix}$$

$$X_{+} = \begin{bmatrix} x_d(k_0 + 1) & \dots & x_d(k_0 + T_2^{(1)}) \end{bmatrix}$$

B. Nortmann, A. Monti, M. Sassano and T. Mylvaganam, “Nash Equilibria for Linear Quadratic Discrete-Time Dynamic Games via Iterative and Data-Driven Algorithms”, IEEE Transactions on Automatic Control, vol. 69, no. 10, pp. 6561-6575, 2024.

Data-driven iterative Nash equilibrium finding

Enables players to iteratively converge to a feedback Nash equilibrium of the game despite having *incomplete system and cost information*

Each *player updates only own strategy* & does not explicitly estimate or communicate strategies of other players.

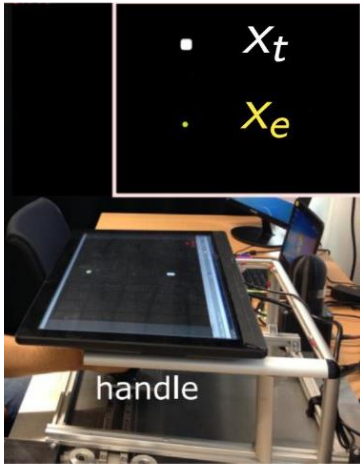
Distributed in the sense that players only require *measurements of state and own inputs*

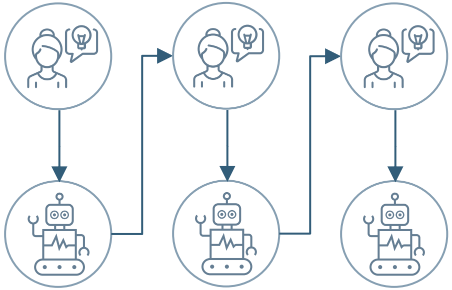
Computational complexity and data requirements for update of each player do *not depend on the total number of players* in game

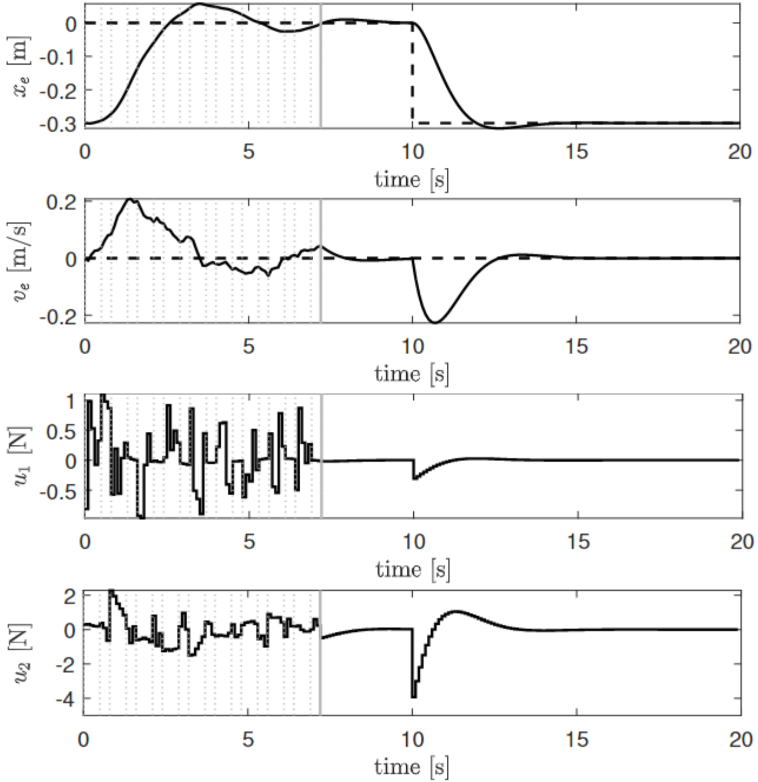
B. Nortmann, A. Monti, M. Sassano and T. Mylvaganam, “Nash Equilibria for Linear Quadratic Discrete-Time Dynamic Games via Iterative and Data-Driven Algorithms”, IEEE Transactions on Automatic Control, vol. 69, no. 10, pp. 6561-6575, 2024.

Learning to interact

Simulation of human operator u_1 and contact robot u_2 interacting for arm reaching movements







B. Nortmann, A. Monti, M. Sassano and T. Mylvaganam, "Nash Equilibria for Linear Quadratic Discrete-Time Dynamic Games via Iterative and Data-Driven Algorithms", IEEE Transactions on Automatic Control, vol. 69, no. 10, pp. 6561-6575, 2024.

Photo: Y. Li, G. Carboni, F. Gonzalez et al., "Differential game theory for versatile physical human-robot interaction," Nature Machine Intelligence, vol. 1, no. 1, pp. 36–43, Jan. 2019.

Thank you!

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IMPERIAL

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NCCR Automation

Empa Materials Science and Technology

FNE solutions of infinite-horizon LQ games are important to model and design dynamic interactions & challenging problems remain

Insights into number and properties of FNE solutions for scalar games

Iterative methods allow us to find (some) FNE solutions

Data-driven methods allow us to overcome incomplete information

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Thank you!