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WASP WALLENBERG AI, AUTONOMOUS SYSTEMS AND SOFTWARE PROGRAM

ETH zürich

NCCR Automation

Aligning the "Socio" in Socio-Technical Control: Trustworthy, Fair, and Efficient Resource Allocation with Karma Economies

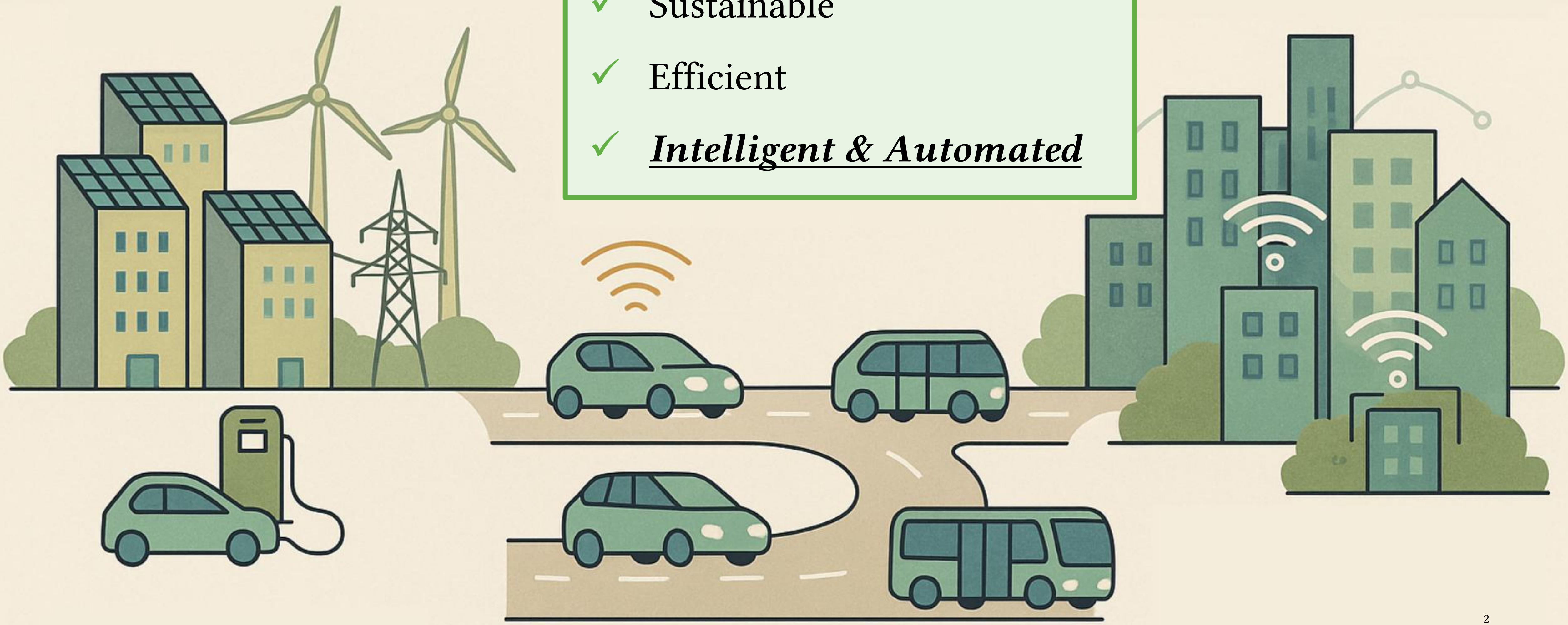
Ezzat Elokda

Game On! Seminar

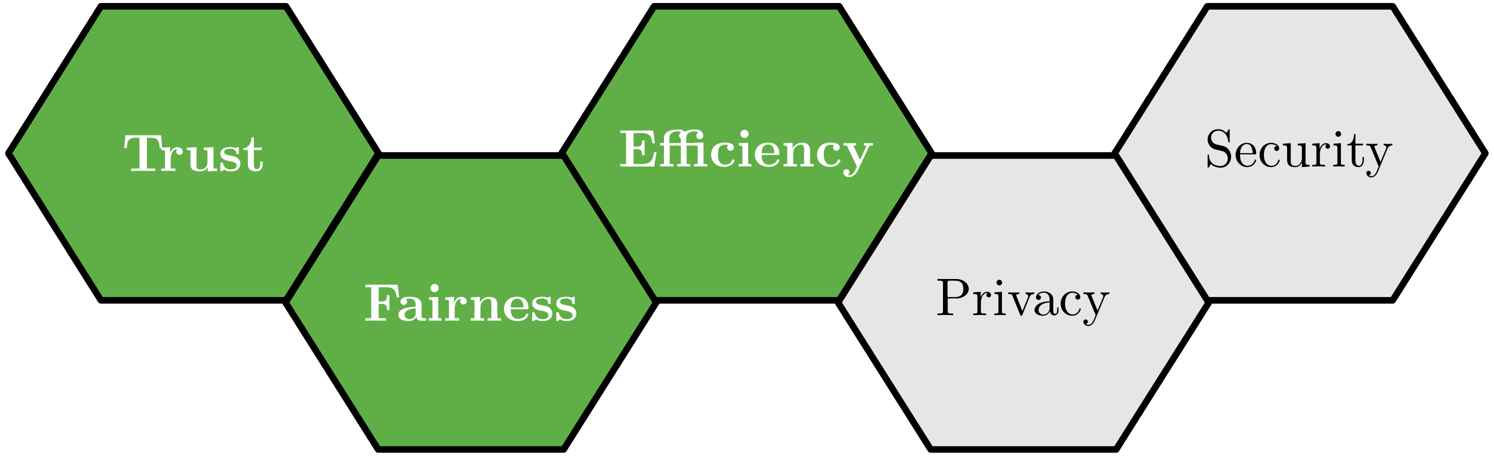
23.06.2026

Utopian Vision of the Future Smart City:

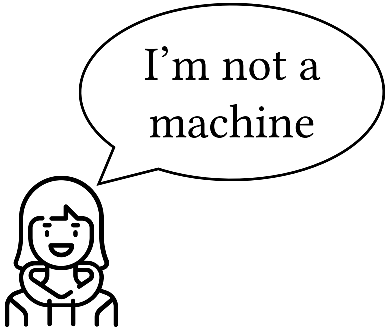
- ✓ Sustainable
- ✓ Efficient
- ✓ ***Intelligent & Automated***



Cyber-Physical-***Human*** Systems: Not all about *"Efficiency"*



… and many more complex human factors



Definitions: (informal)

- *Trust* = transparency; acting in human's best selfish interest
- *Fairness* = complex to define; but important
- *Efficiency* = also complex; for now: ability to tailor to human's private preferences

Conventional Wisdom: *Trust* + *Fairness* + *Efficiency* incompatible

Hylland, A. Strategy proofness of voting procedures with lotteries as outcomes and infinite sets of strategies. *Technical Report* (1980)

Nisan, N. *et al.* Introduction to mechanism design (for computer scientists). *Algorithmic Game Theory* (2007)

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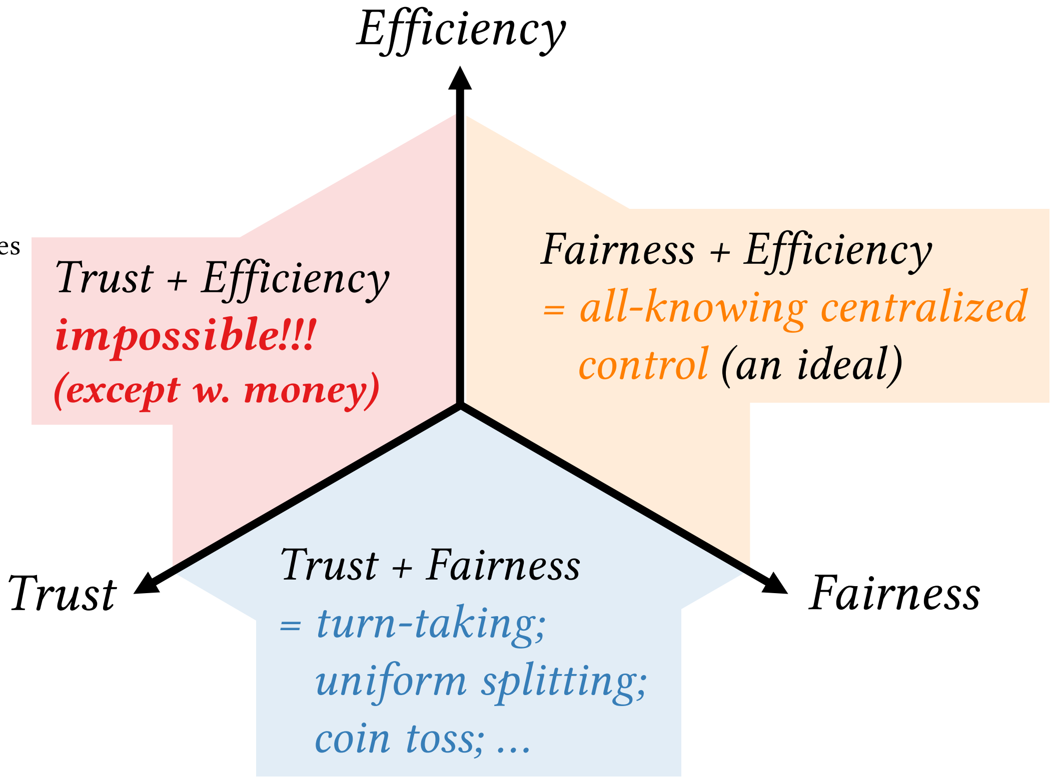

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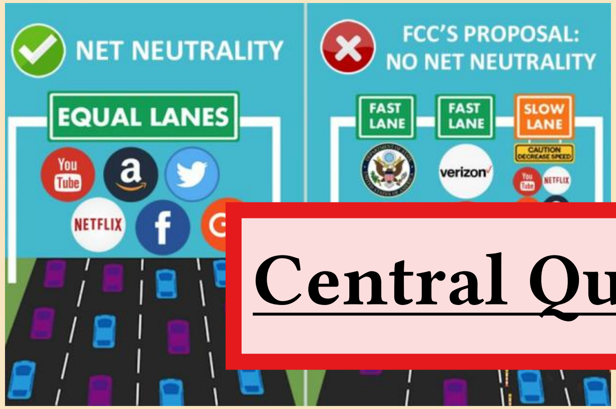
Monetary Socio-Technical Control is Controversial!

… to say the least

Money ***okay!***

Data Networks



© The Mirror, The Battle Over Net Neutrality

Transportation

© USA Today, Dynamic tolls: How highways can charge $40 for driving 10 miles

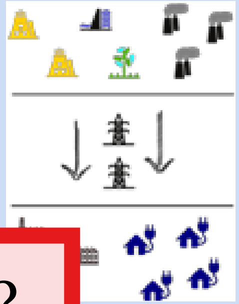
drive into after $9 congestion toll ups driver costs to $27

Manhattan Congestion Pricing Hits a Nerve as Hearings Open

- MTA hosts first of six virtual public hearings for input
- Plan calls for charging some motorists as much as $23 in tolls

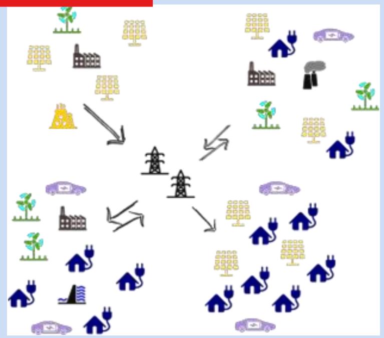
© Bloomberg

Energy



Present

Central Question: Can we get rid of money?



Future?

© Pierre Pinson, Renewables in Electricity Markets

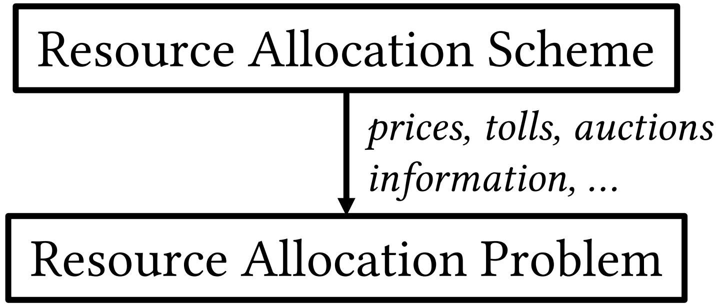
Money ***no!***

Public sentiment:

Money ***no!*** *Money* ***okay!*** *Money* ***no!*** *Money* ***okay!***

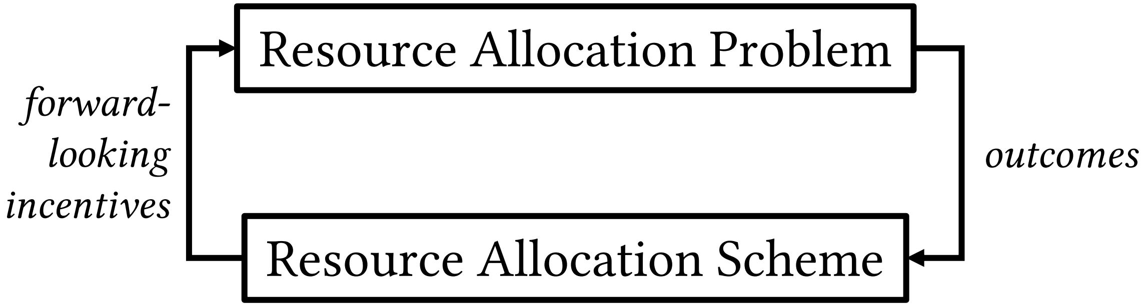
Control perspective to the rescue!

Traditional “Open-Loop” View



- Single-shot and static
- Myopic fairness / efficiency / social welfare
- Trust + fairness + efficiency in tension

“Closed-Loop” View



- Repetitive and dynamic
- Long-run fairness / efficiency / social welfare
- Trust + fairness + efficiency compatible
- *(also embraces that society evolves and adapts)*

This talk: The power of adding a state variable to resource allocations

… humans need data, roads, energy… many many times, for the indefinite future

Karma

"Today me, tomorrow thee"

Dynamic Population Game (DPG) Model

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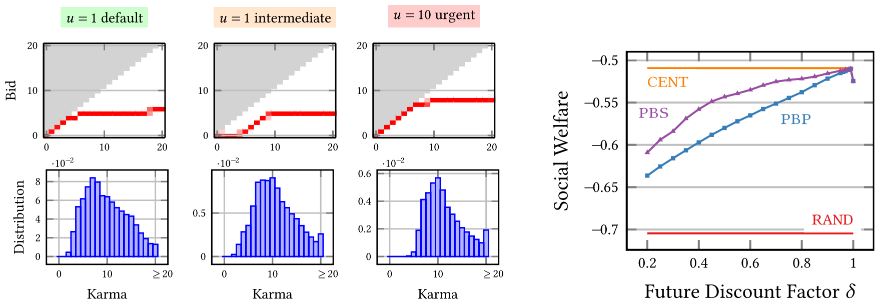
Theorem: Existence of Karma SNE



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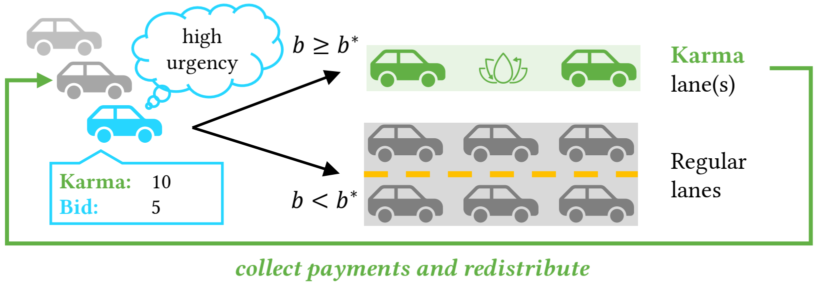

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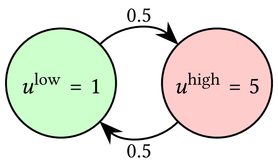

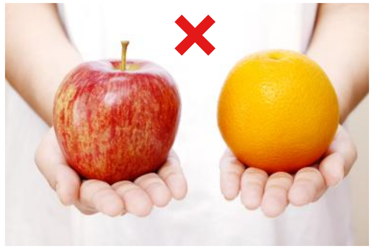

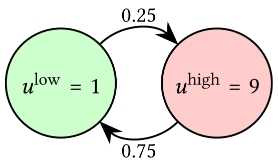

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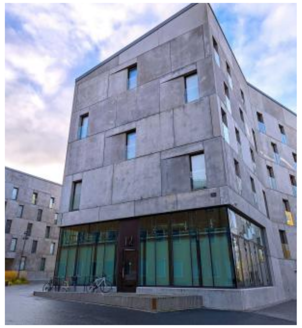
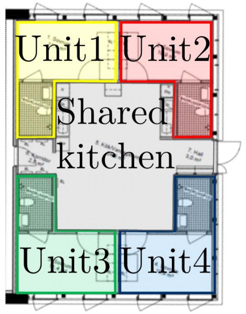
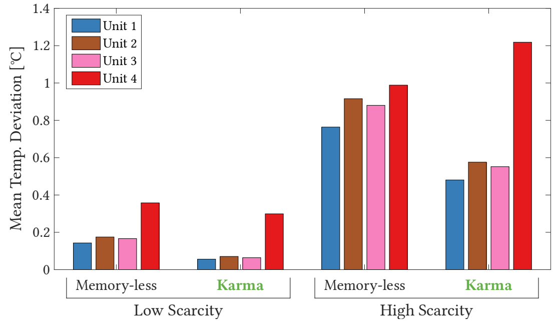
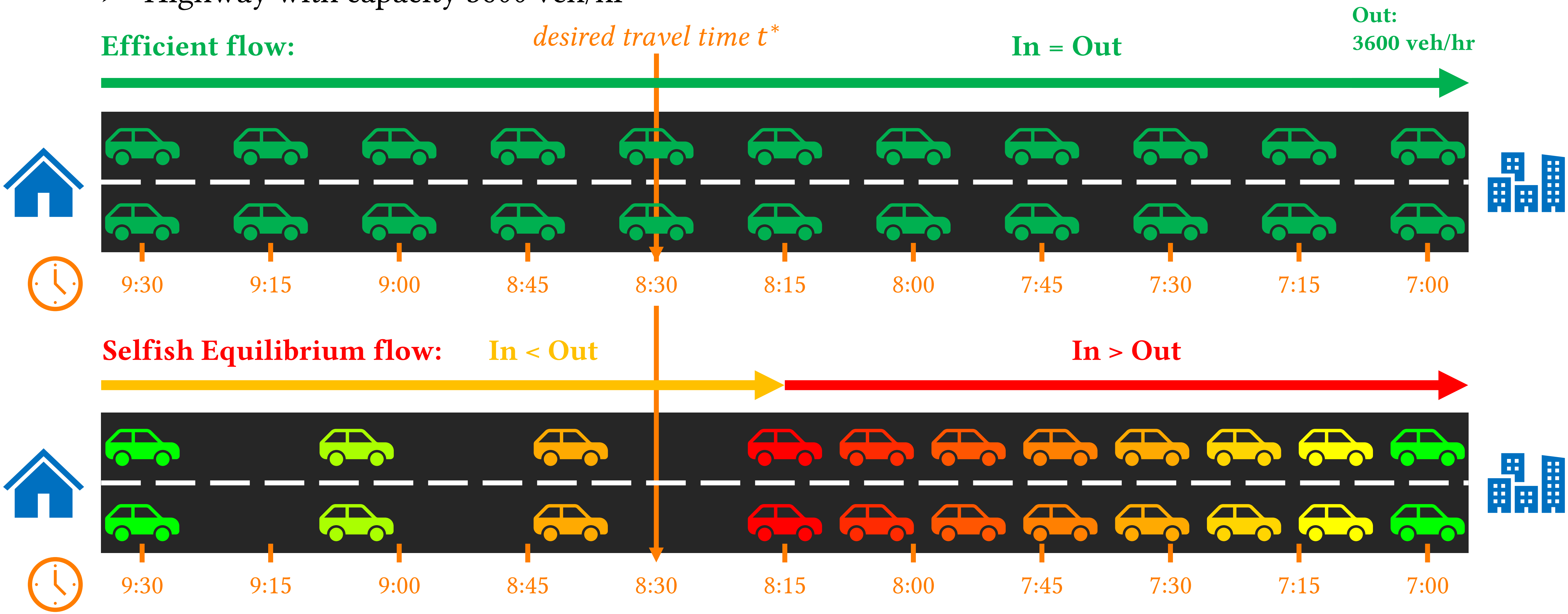






Review: Traffic Bottleneck Congestion Model

- 9000 commuters …
- Highway with capacity 3600 veh/hr



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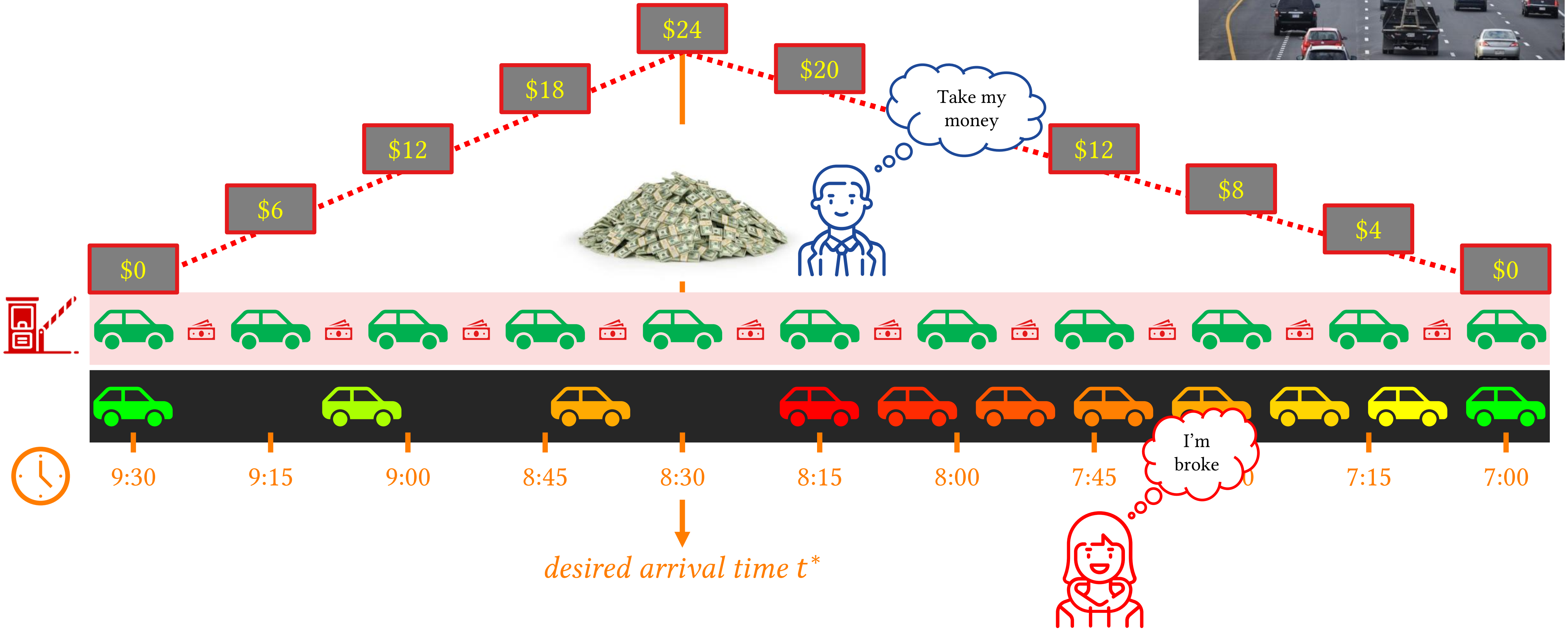
Review: Classical Monetary Solution

Tolled priority lanes



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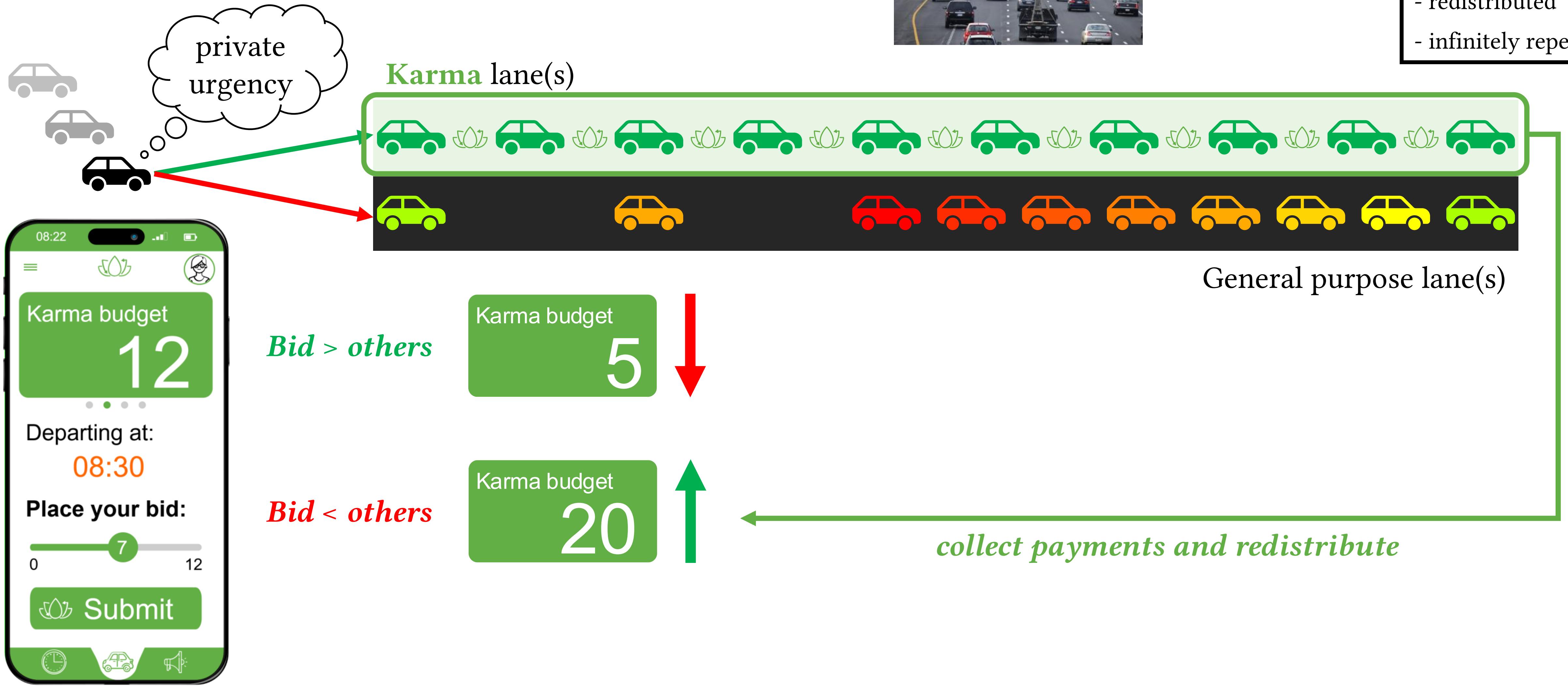
Karma Economy: Demonstration

karma

- non-tradable credit
- used for bidding
- redistributed
- infinitely repeated



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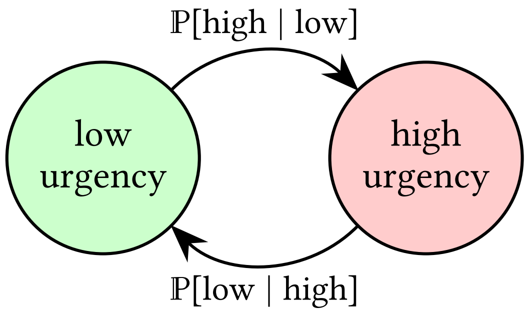
Individual ***urgency*** is *dynamic* over days!

"I woke up early and have no meetings"



"I woke late and have an important meeting"

Standing Assumption: Urgency dynamics are:

1. *Exogenous*;
2. *Independent* across users;
3. *Private* to each user;
4. Grouped in a finite number of *urgency types;*
5. Representable per type by a *finite-state Markov chain.*

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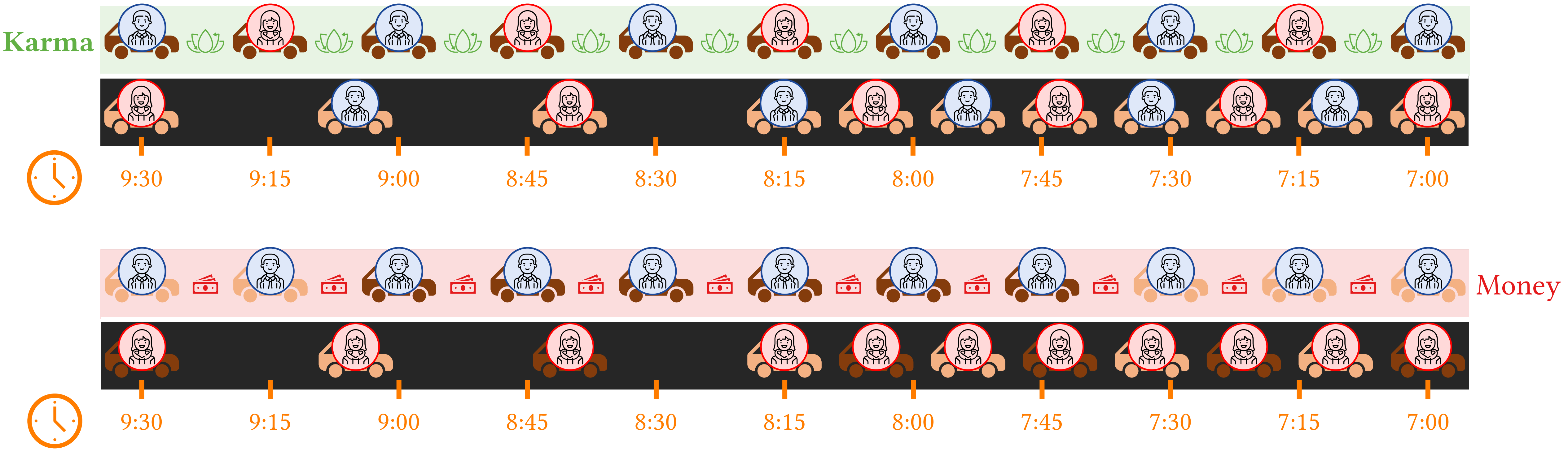
Karma vs. Money : *Stationary Nash Equilibrium (SNE)* predictions

Urgency Process		Willingness to Pay	
	Not in a rush (50%)	$10/hr saved	$1/hr saved
	In a rush (50%)	$50/hr saved	$5/hr saved



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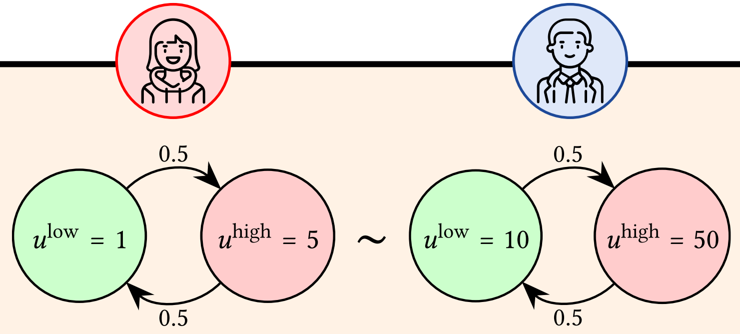
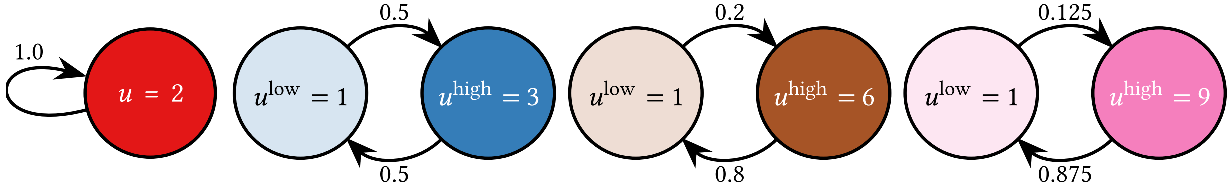
Formal Fairness + Efficiency Results

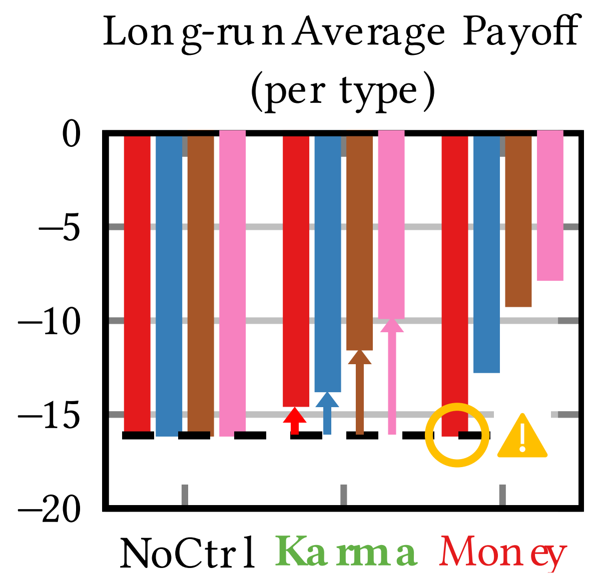
Theorem (Equity under Heterogenous Income).
Karma SNE are scale-invariant (in the no future discounting limit $\delta \to 1$):

$$\lim_{\delta \to 1}\left[\frac{\text{long–run average payoff}}{\text{long–run average urgency}} - \frac{\text{long–run average payoff}}{\text{long–run average urgency}}\right] = 0$$



Theorem (Strong Pareto Improvement).
In Karma SNE, all user types are guaranteed <u>strict</u> benefits over *No Control.*
(under general urgency process heterogeneity)





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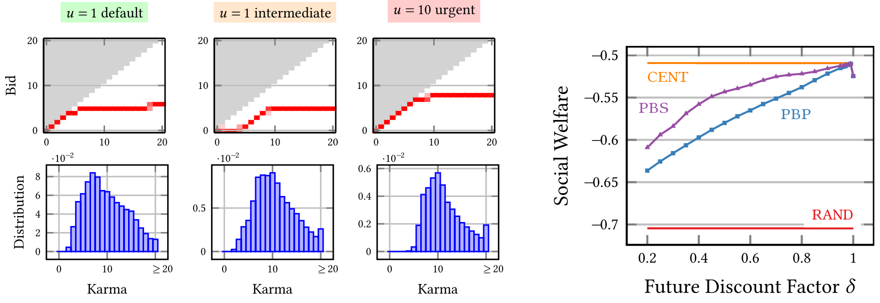

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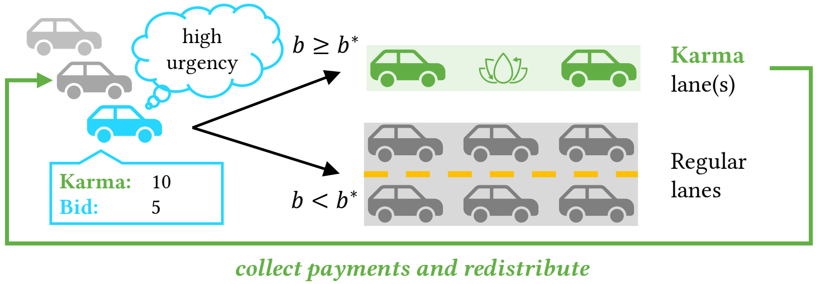

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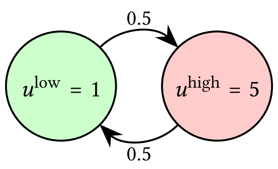

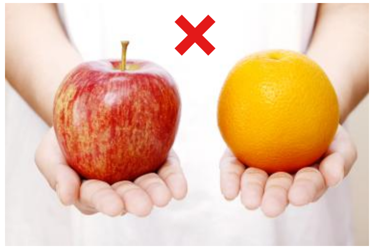

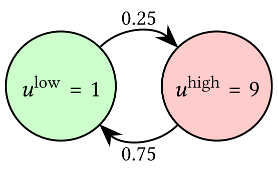

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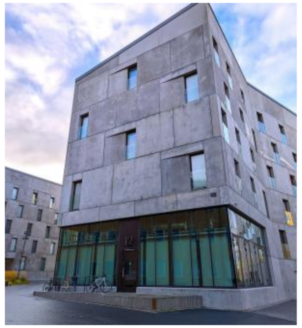
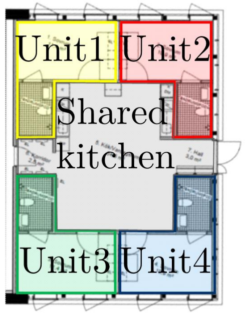
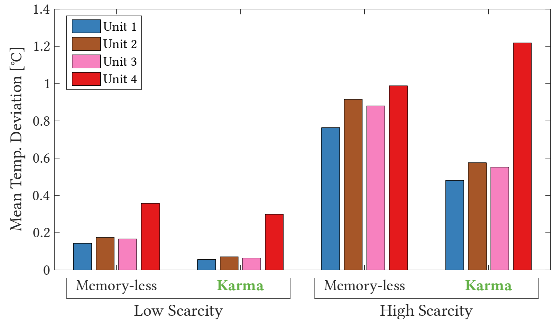






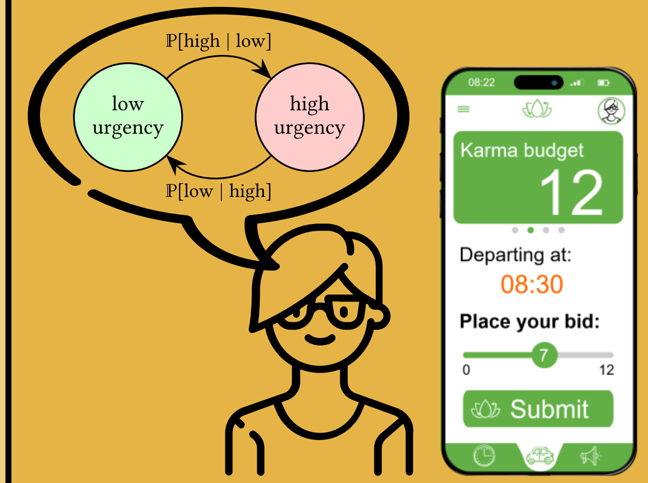
Dynamic Population Game (DPG) Model of a Karma Economy
(a.k.a. mean-field game model)

Notation

$[\cdot]$	discrete argument
$(\cdot)$	continuous argument

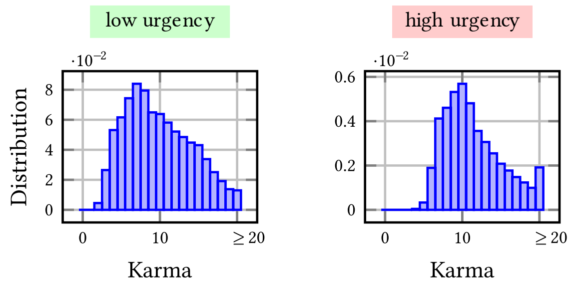
Individual State/Action

- **Private State** $x = [u, k]$
  - Urgency: $u \in \{u^1, \dots, u^{n_u}\}$
  - Karma: $k \in \{0, \dots, k^{\max}\}$
- **Action** $a = [b, t^d]$
  - Karma bid: $b \leq k \in \mathbb{N}$
  - Departure time: $t^d \in \{t^{d,1}, \dots, t^{d,n_t}\}$

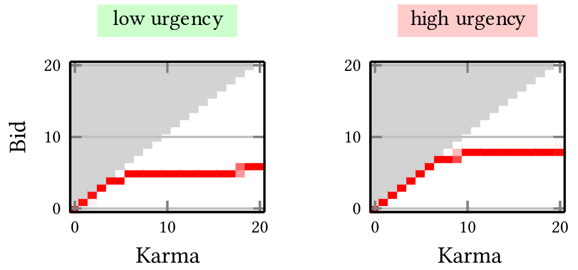


Social State $(\boldsymbol{d}, \boldsymbol{\pi})$

- **State distribution** $\boldsymbol{d}$
  - Proportion of state x: $d[x]$

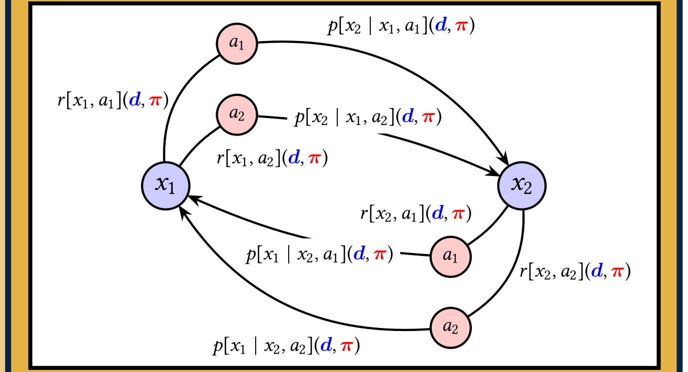


- **Behavioral policy** $\boldsymbol{\pi}$
  - Action probability given state: $\pi[a|x]$



Individual Markov Decision Process (MDP)

- **Immediate payoff**
  $r[x, a](\boldsymbol{d}, \boldsymbol{\pi})$
- **State transition probabilities**
  $p[x^+ \mid x, a](\boldsymbol{d}, \boldsymbol{\pi})$



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Stationary Nash Equilibrium (SNE)

... and its existence

Notation	
d^*	SNE state distribution
π^*	SNE policy
(d^*, π^*)	SNE social state
P	state transition matrix
B_δ	δ-discounted optimal policies

Definition (Stationary Nash Equilibrium (SNE))**.** Social state (d^*, π^*):

1. State distribution stationary: $d^* = P(d^*, \pi^*)^\top d^*$
2. Policy δ-discounted best response: $\pi^* \in B_\delta(d^*, \pi^*)$

Assumption (Continuity)**.** Immediate payoff & state transition functions continuous in social state.

Assumption (Karma Preservation). The system average karma is preserved.

Theorem (Existence of Karma SNE)**.**
For each $\bar{k} \in \mathbb{N}$, a Karma SNE with system average karma $\bar{k}$ is guaranteed to exist.

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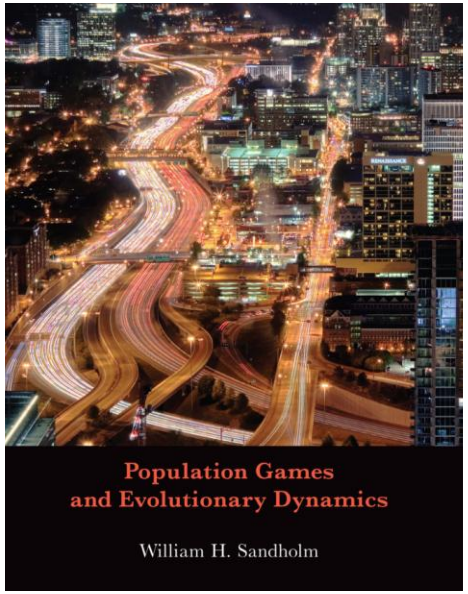
Reduction SNE $\longrightarrow$ NE

Theorem:

For every ***Dynamic Population Game*** $\mathcal{G}$, …

… there exists a (static) ***Population Game*** $F^{\mathcal{G}}$,

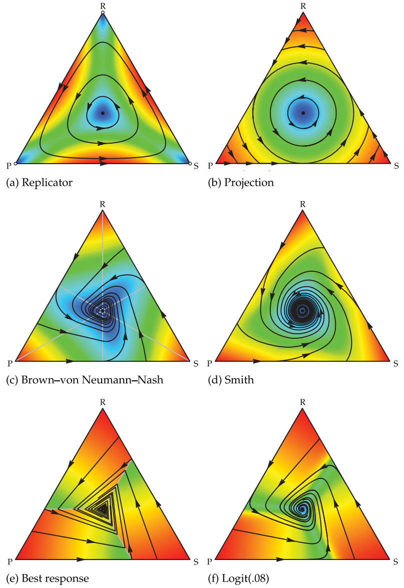
… whose ***Nash Equilibria*** coincide with the ***Stationary Nash Equilibria*** of $\mathcal{G}$.



Proof idea:

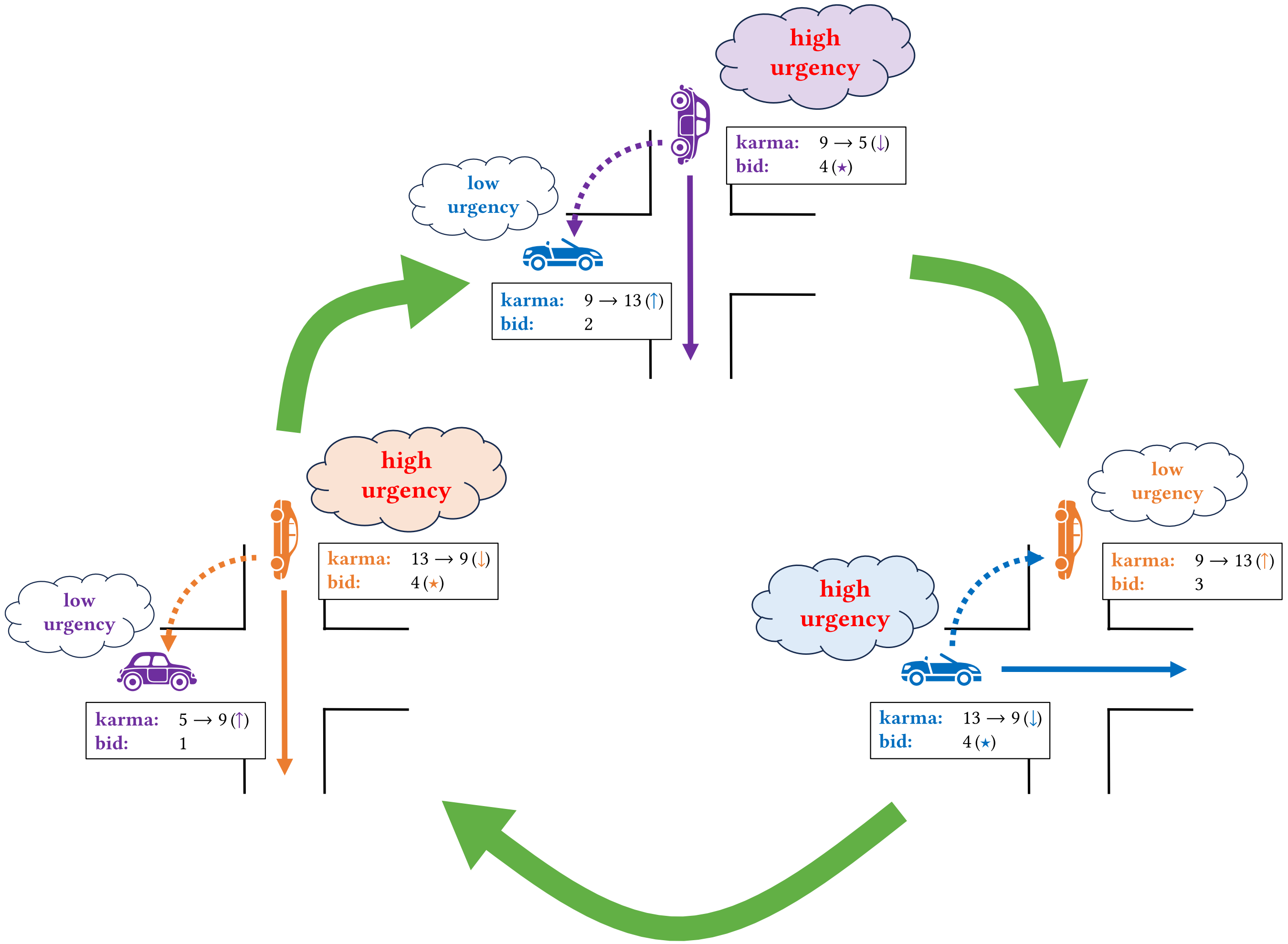
- Use *state-action value function* as payoffs in $F^{\mathcal{G}}$
- Augment $F^{\mathcal{G}}$ with population *'playing the dynamics'*

Evolutionary dynamics
For SNE computation



Example 1: Anonymous pairwise interactions

State:	$x = [u, k]$
➢ Urgency	$u \in \{u^1, \dots, u^{n_u}\}$
➢ Karma	$k \in \{0, \dots, k^{\max}\}$
Action:	$a = b$
➢ Karma bid	$b \leq k \in \mathbb{N}$



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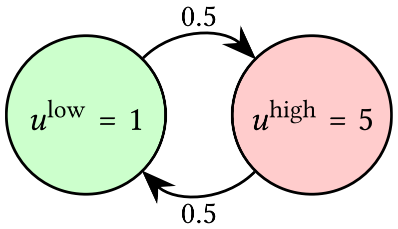
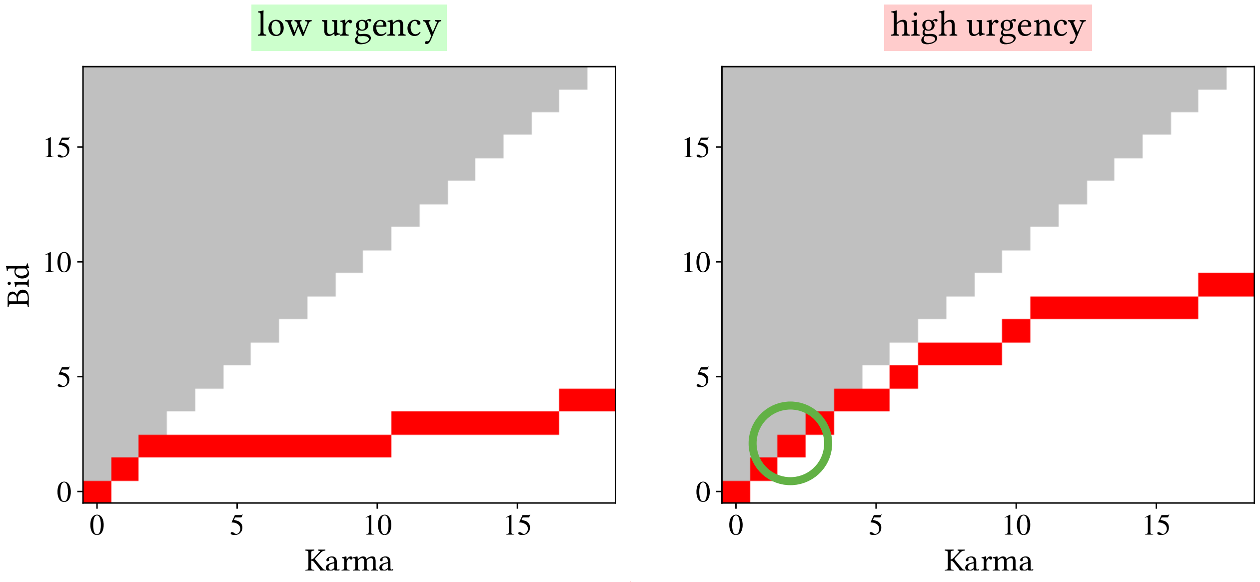
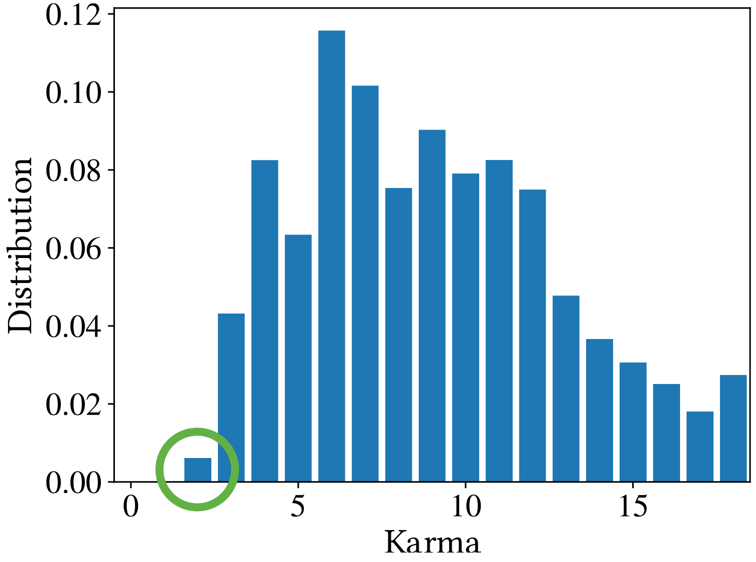
Example 1: Anonymous pairwise interactions

Karma SNE with future discount factor $\delta = 0.98$ *and urgency process:*





Policy $\boldsymbol{\pi}^*$



Stationary karma distribution $\boldsymbol{d}^*$

➢ Karma SNE almost fully efficient (very low probability that low urgency outbids high urgency)

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Example 2: Priority Lanes Allocation

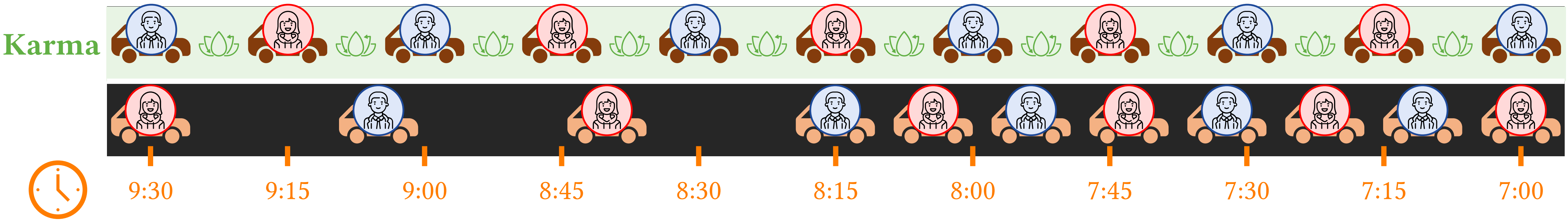
Urgency Process		Willingness to Pay	
	Not in a rush (50%)	$10/hr saved	$1/hr saved
	In a rush (50%)	$50/hr saved	$5/hr saved

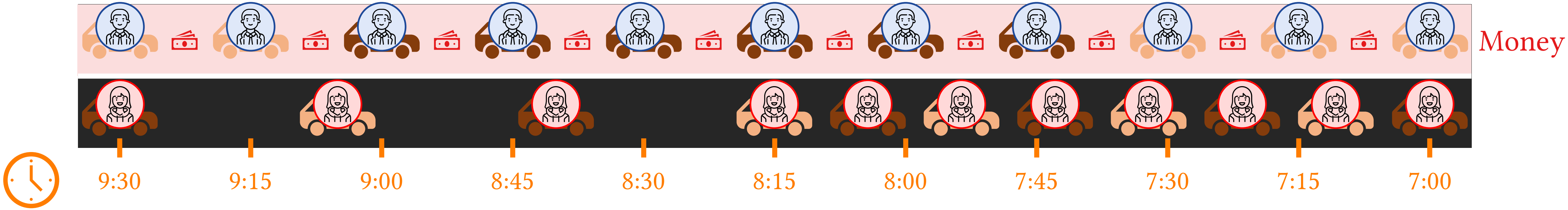
State: $x = [u, k]$

- Urgency $u \in \{u^1, \dots, u^{n_u}\}$
- Karma $k \in \{0, \dots, k^{\max}\}$

Action: $a = [b, t]$

- Karma bid $b \leq k \in \mathbb{N}$
- Dep. time $t^d \in \{7{:}00, 7{:}15, \dots\}$





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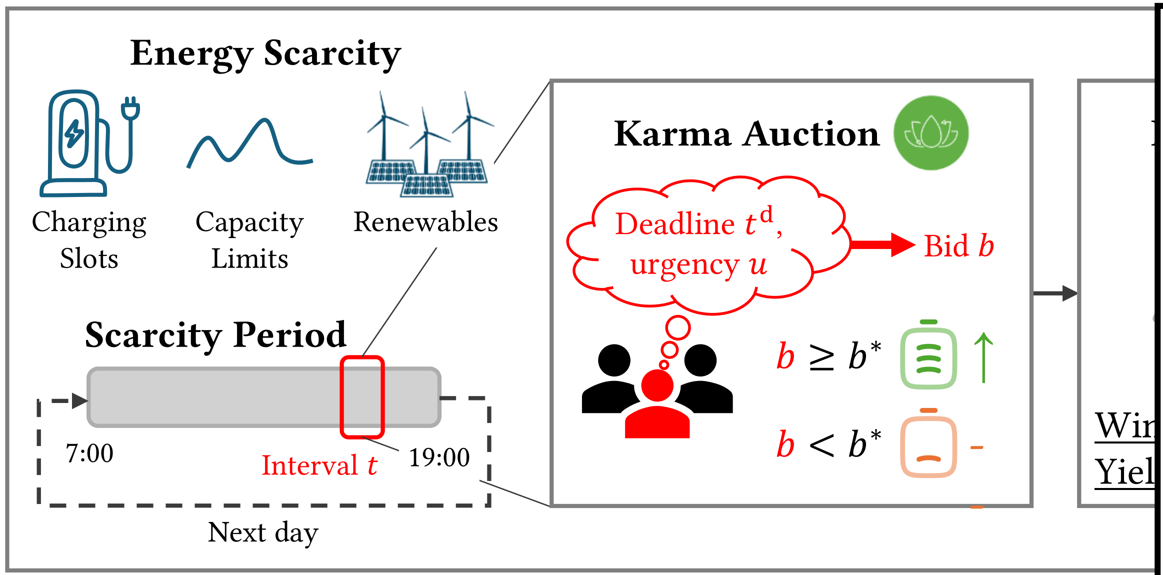
Example 3: Flexible Electric Vehicle (EV) Charging



State: $x = [t, l, s, s^{\mathrm{d}}, t^{\mathrm{d}}, u, k]$

- Time of day $t \in \{7{:}00, 7{:}30, \dots\}$
- Presence state $l \in \{\text{arriving}, \text{present}, \text{departed}\}$
- State of Charge (SOC) $s \in \{8\text{kWh}, 16\text{kWh}, \dots\}$
- Desired SOC $s^{\mathrm{d}} \in \{32\text{kWh}, 48\text{kWh}, 64\text{kWh}\}$
- Deadline to charge $t^{\mathrm{d}} \in \{30\text{min}, 1\text{hr}, \dots\}$
- Urgency $u \in \{u^1, \dots, u^{n_u}\}$
- Karma $k \in \{0, \dots, k^{\max}\}$

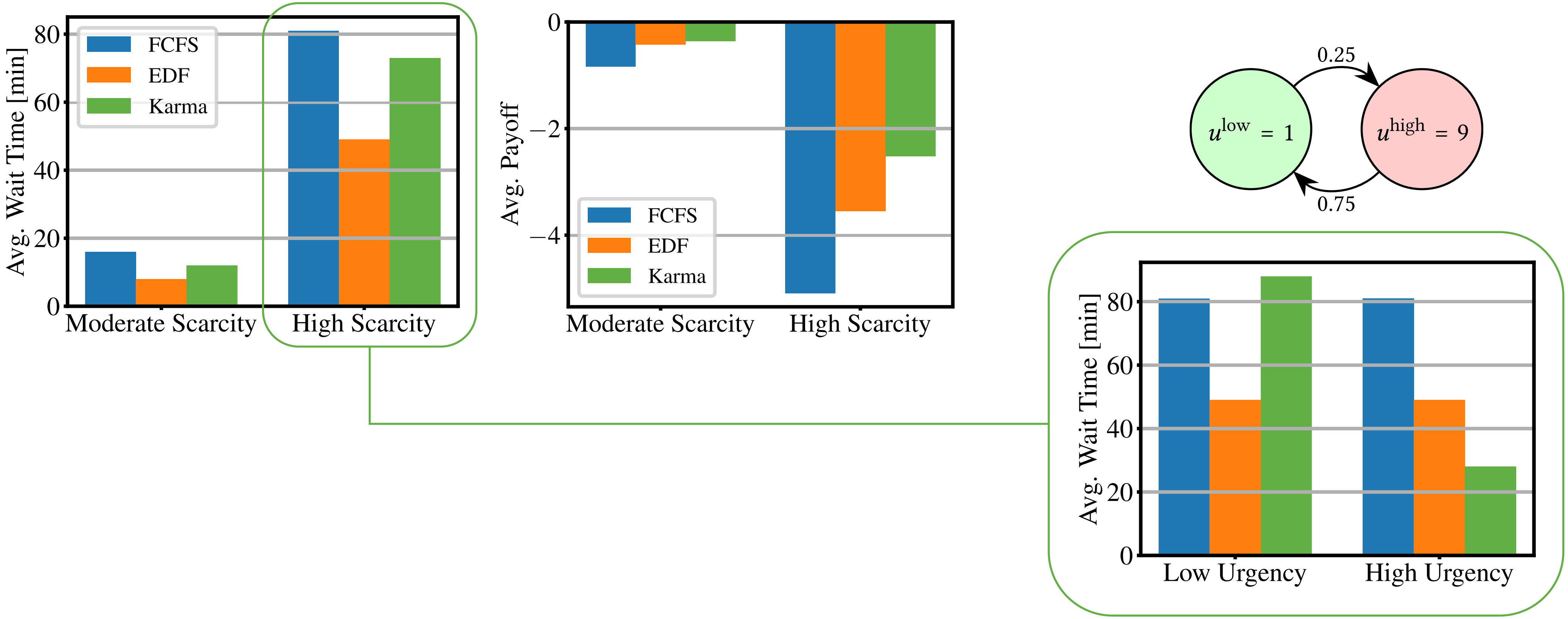
Action: $a = b$

- Karma bid $b \leq k \in \mathbb{N}$

Elokda, E., Wang, R., Johansson, K. H., Fontan, A. Flexible Electric Vehicle Charging with Karma. *CDC* (submitted, 2026)

Example 3: Flexible Electric Vehicle (EV) Charging

Karma SNE vs. First-Come-First-Serve (FCFS) vs. Earliest-Deadline-First (EDF)



➢ <u>Karma SNE balances between meeting deadlines and prioritizing high urgency</u> (according to private deadlines + urgency)

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Dynamic Population Game (DPG) Model

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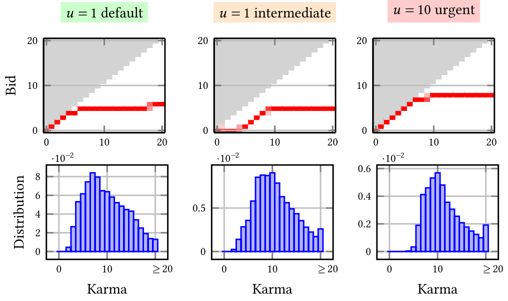

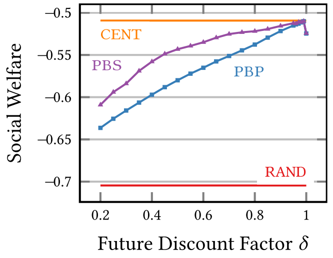

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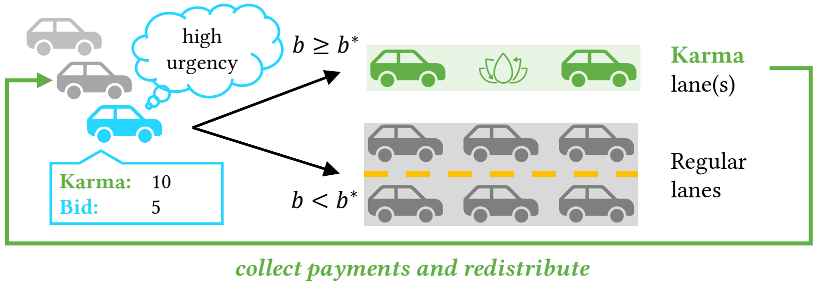

Karma

"Today me, tomorrow thee"

What is *Fairness* and *Efficiency*?

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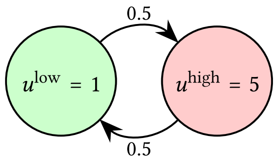
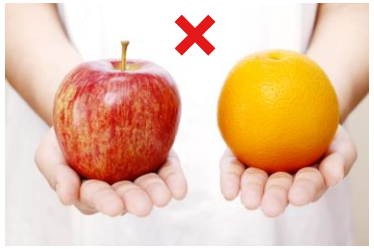
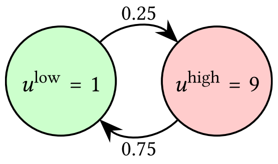
Theorem: KE is MLNW







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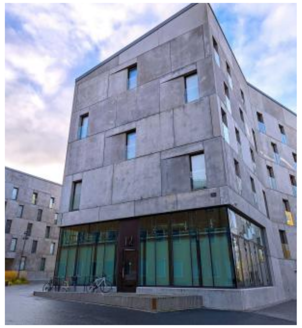
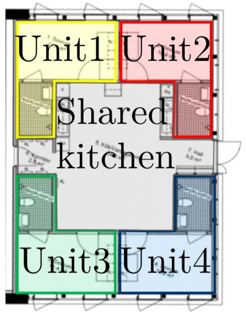
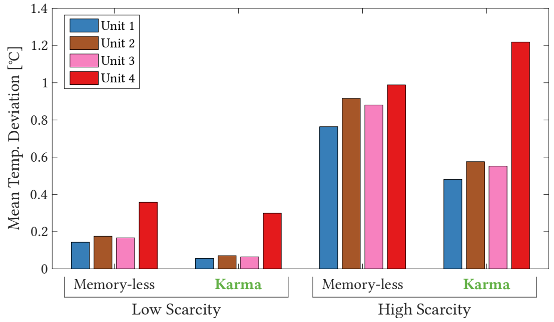


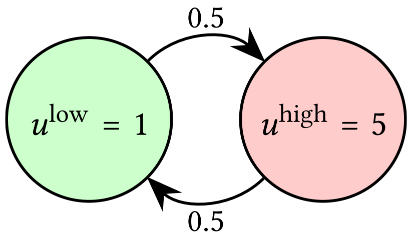




What is ***Fair and Efficient***? – *Let’s break it down*

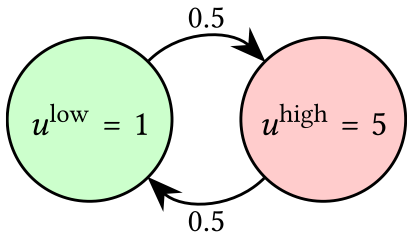
Notation	
$\boldsymbol{\psi}$	Long-run allocation
$\boldsymbol{\psi}^{\mathbf{0}}$	Benchmark long-run allocation
$\overline{r}_i$	User i’s long-run average payoff

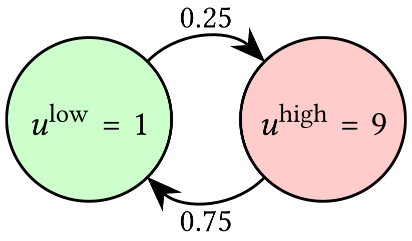
Homogeneous Urgency Processes



Fair and Efficient = prioritize high urgency
(everyone gets their turn for high urgency)

Heterogeneous Urgency Processes





Fair and Efficient = prioritize high urgency
in interpersonally non-comparable manner

Embodied by ***Long-run Nash Welfare (LNW)****!*

$$\mathrm{LNW}(\boldsymbol{\psi}) = \prod_i \left(\overline{r}_i(\boldsymbol{\psi}) - \overline{r}_i(\boldsymbol{\psi}^0)\right)$$

long-run allocation $\boldsymbol{\psi}$*:*
maps urgency → probabilistic allocation

Properties of (classical) *Nash Welfare*:

- Unique social welfare function satisfying basic efficiency axioms without interpersonal comparisons
- Maximized by Nash bargaining solution
(regarded as fair/dynamically stable bargaining outcome)

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Centralized *Maximum Long-run Nash Welfare (MLNW)* Problem

Notation

$[\cdot]$	discrete argument
$(\cdot)$	continuous argument

- Players $i \in \mathcal{N}$, with *weights / access rights / entitlements* $w_i \in \mathbb{R}_+$
- Resources $j \in \mathcal{M}$, with capacities $c_j \in \mathbb{R}_+$
- Individual urgency $u_i \in \{u_i^1, \dots, u_i^{m_i}\}$, with stationary probabilities $\sigma_i[u_i] \in [0,1]$
- Elementary rewards $r_{i,j} \in \mathbb{R}$ (for allocating j to i), benchmark elementary rewards $r_{i,j}^0 \in \mathbb{R}$
- <u>*Long-run allocations*</u> $\psi_{i,j}[u_i] \in [0,1]$ (prob. of allocating j to i when in u_i)

$$
\begin{aligned}
&\max_{\boldsymbol{\psi}} \sum_i w_i \log\big(\bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0)\big) \\
\text{s.t.:}\quad & \bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0) = \sum_{u_i} \sigma_i[u_i]\, u_i \sum_j \psi_{i,j}[u_i]\big(r_{i,j} - r_{i,j}^0\big) && \forall i \\
& \sum_i \sum_{u_i} \sigma_i[u_i]\, \psi_{i,j}[u_i] \le c_j, && \forall j \\
& 0 \le \psi_{i,j}[u_i] \le 1, && \forall i, u_i, j
\end{aligned}
$$

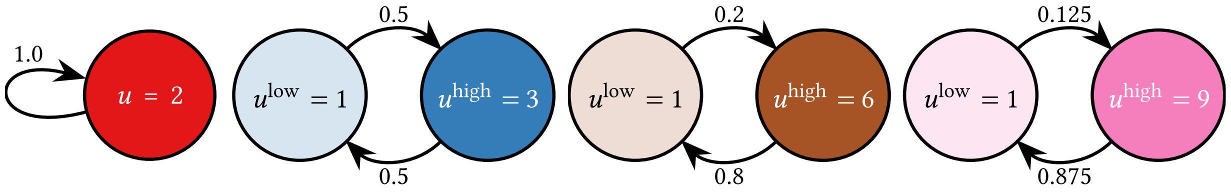
equivalent to $\max \prod_i \big(\bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0)\big)^{w_i}$

long-run expected average payoff improvements

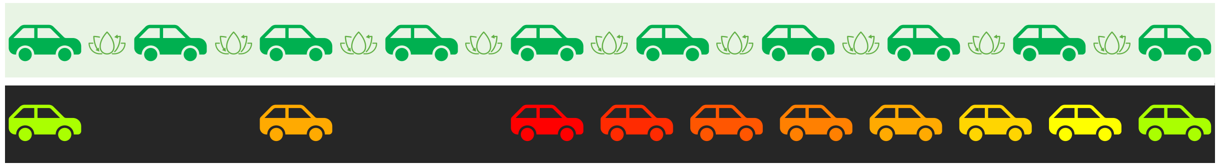
resource constraints (satisfied in expectation)

probabilistic allocations

MLNW in Action

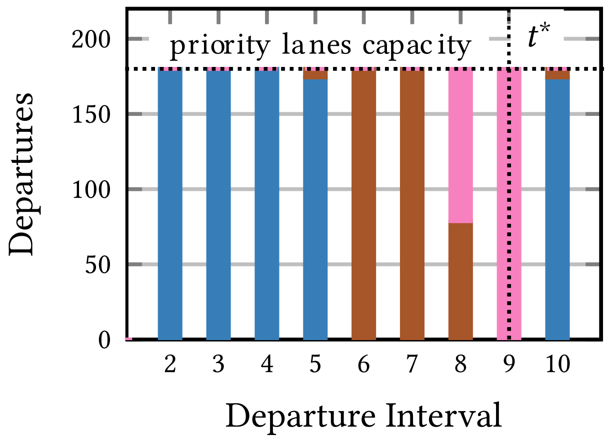


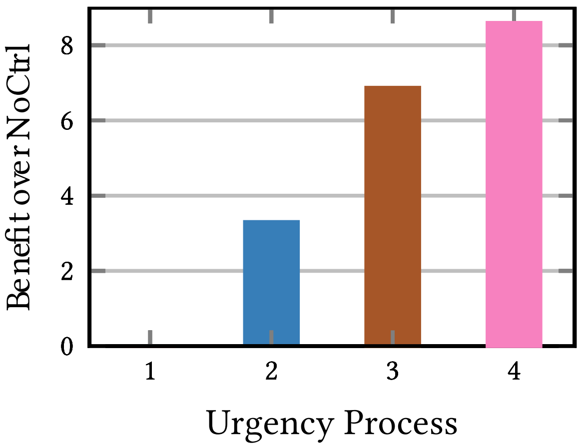
Karma lane(s)



General purpose lane(s)

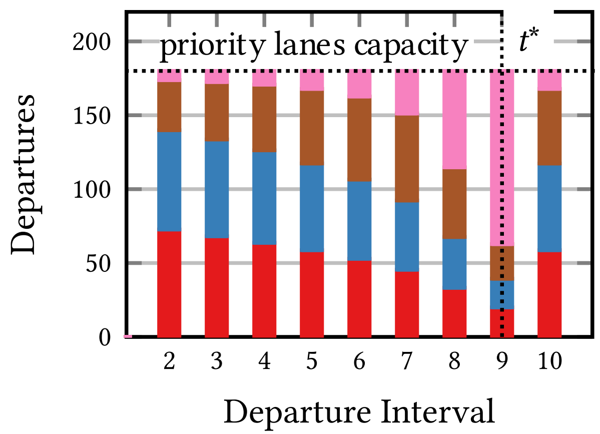
Maximum Utilitarian (Sum) Welfare

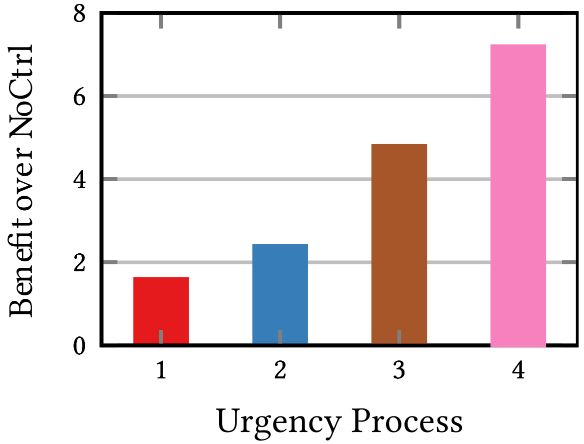




✗ some players never get priority
✗ sensitive to scaling
✗ (incorrectly) assumes interpersonal comparability

Maximum Long-run Nash Welfare





✓ urgency dynamics-tailored *"trading of resource shares"*
✓ scale invariant + strong Pareto improving
$\Longrightarrow$ formalizes *Fairness + Efficiency*

MLNW versus (relaxed) Karma Stationary Nash Equilibrium (SNE)

<u>Centralized Maximum Long-Run Nash Welfare (MLNW) Problem</u>

$$\max_{\boldsymbol{\psi}} \sum_i w_i \log\left(\bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0)\right)$$

$$\text{s.t.:} \quad \bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0) = \sum_{u_i} \sigma_i[u_i]\, u_i \sum_j \psi_{i,j}[u_i]\left(r_{i,j} - r_{i,j}^0\right) \qquad \forall i$$

$$\sum_i \sum_{u_i} \sigma_i[u_i]\, \psi_{i,j}[u_i] \leq c_j, \qquad \forall j$$

$$0 \leq \psi_{i,j}[u_i] \leq 1, \qquad \forall i, u_i, j$$

<u>Individual Karma SNE Problems for each $i \in \mathcal{N}$</u>

$$\max_{\boldsymbol{\psi}_i} \bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0)$$

$$\text{s.t.:} \quad \bar{r}_i(\boldsymbol{\psi}) - \bar{r}_i(\boldsymbol{\psi}^0) = \sum_{u_i} \sigma_i[u_i]\, u_i \sum_j \psi_{i,j}[u_i]\left(r_{i,j} - r_{i,j}^0\right)$$

$$\boxed{\sum_{u_i} \sigma_i[u_i] \sum_j \psi_{i,j}[u_i]\, b_j^* \leq \frac{w_i}{\sum_{i'} w_{i'}} \sum_j b_j^*\, c_j\,,}$$

$$0 \leq \psi_{i,j}[u_i] \leq 1, \qquad \forall u_i, j$$

relaxed karma budget constraint:
- *b_j^* stationary resource "clearing bid"*
- *LHS: long-run expected average expenditures*
- *RHS: average gains from (weighted) redistribution*

Are both problems the same??

MLNW is a Karma Equilibrium (KE) *... sometimes!*

Notation

$\boldsymbol{\sigma}_i$	User i's stationary urgency distribution
c_j	Resource j's capacity
$\eta_{i,j}[u_i]$	Lagrange-multiplier for constraint $\psi_{i,j}[u_i] \leq 1$

Definition (Karma Equilibrium (KE); relaxes Karma SNE). Long-run allocation $\boldsymbol{\psi}^*$ + stationary bids $\boldsymbol{b}^*$:

1. *Individual Optimality:* $\boldsymbol{\psi}_i^*$ optimal for player i given clearing bids $\boldsymbol{b}^*$
2. *Resource Clearing:* $b_j^* > 0 \Rightarrow$ resource j allocated at capacity
3. *Budget Balance:* Long-run average karma spent = long-run average karma gained (per player)

$$\sum_{u_i} \sigma_i[u_i] \sum_j \psi_{i,j}^*[u_i]\, b_j^* = \frac{1}{N} \sum_j b_j^*\, c_j$$

Assumption (i.i.d. Urgency)**.** Each user's urgency is independent and identically distributed (across time).

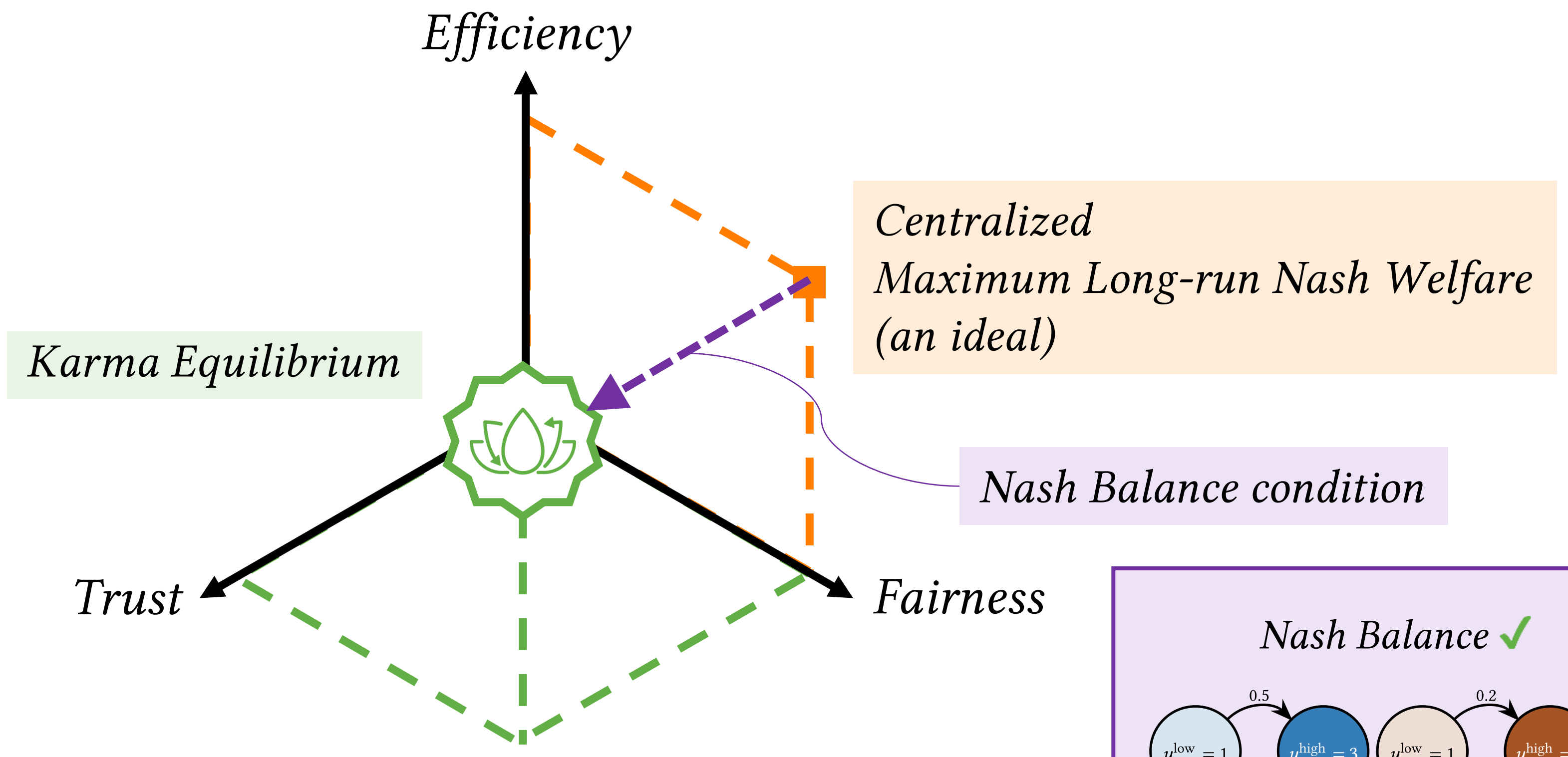
Assumption (Nash Balance). There exists a MLNW solution penalizing users equally for prob. constraints

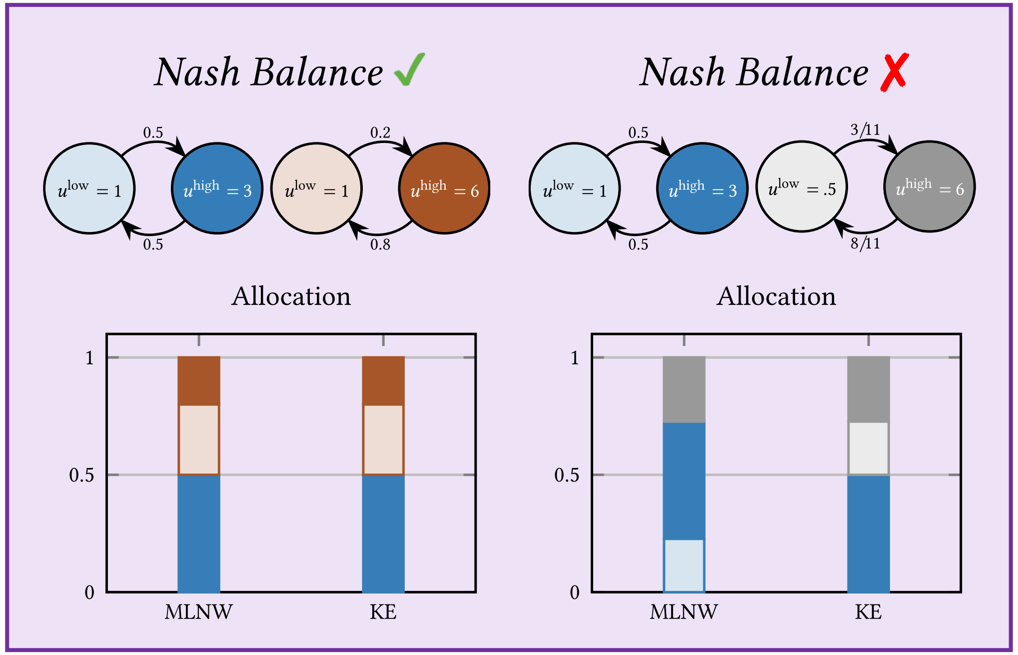
$$\sum_{u_i} \sum_j \eta_i^*[u_i, j] = \sum_{u_{i'}} \sum_j \eta_{i'}^*[u_{i'}, j] = C$$

⇓

Theorem (MLNW is KE)**.** A KE that maximizes Long-run Nash Welfare (LNW) is guaranteed to exist.

Interpretation: *Trust* + *Fairness* + *Efficiency* compatible with karma





Definitions:

- *Trust* = Approximate Bayesian-Nash Incentive Compatibility
- *Fairness* + *Efficiency* = Long-run Nash Welfare (LNW)

Dynamic Population Game (DPG) Model

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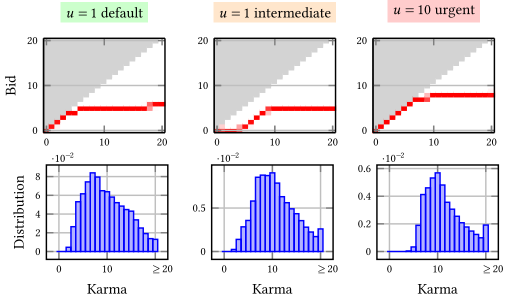

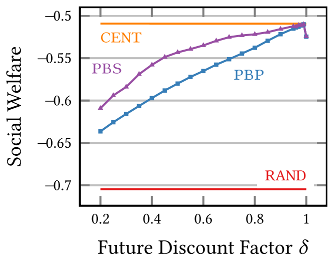

Applications: Traffic Management, EV Charging, ...

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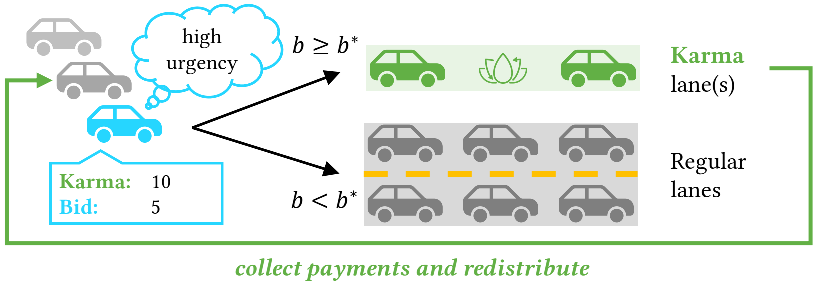

Karma

"Today me, tomorrow thee"

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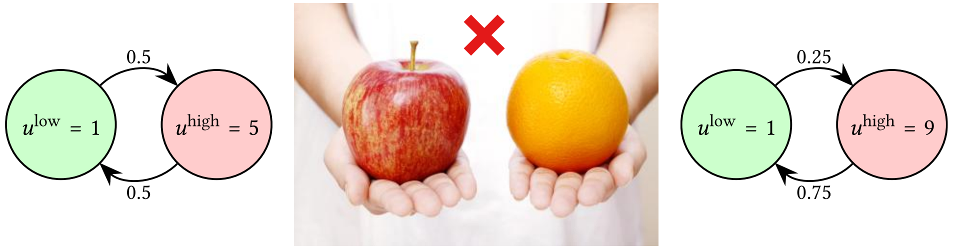

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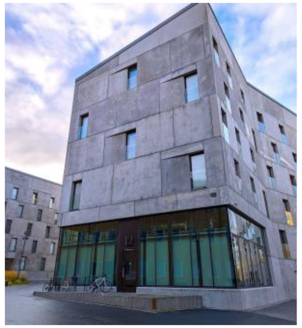
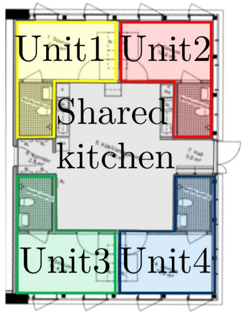
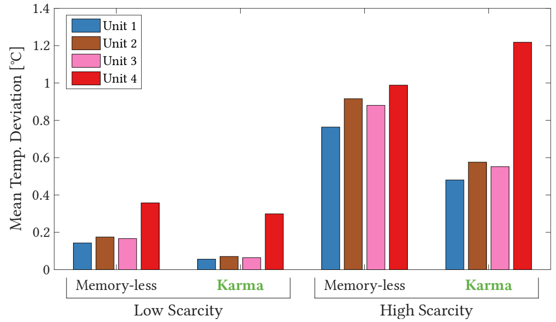
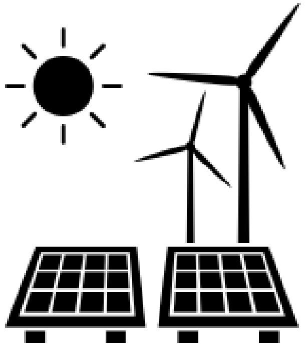






Roadmap to Net-Zero Simplified

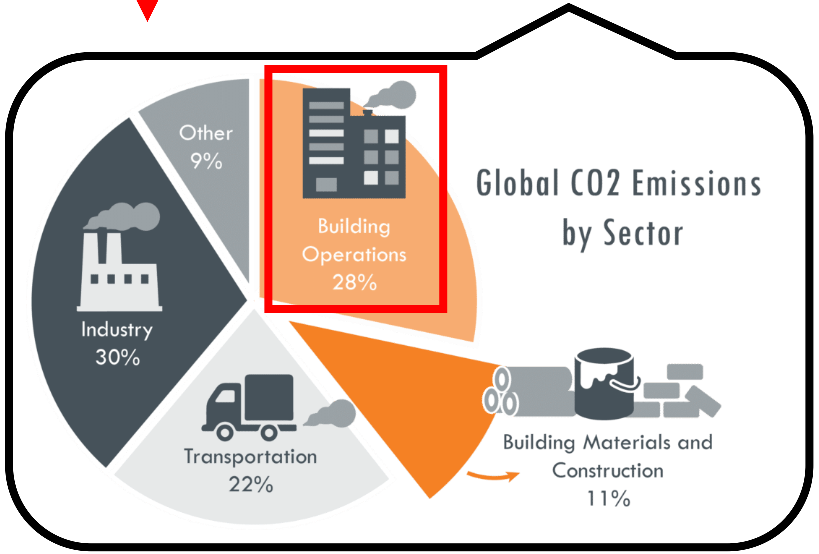
<u>Supply side</u>



ramp up renewable production

promote sustainable behaviors

<u>Demand side</u>



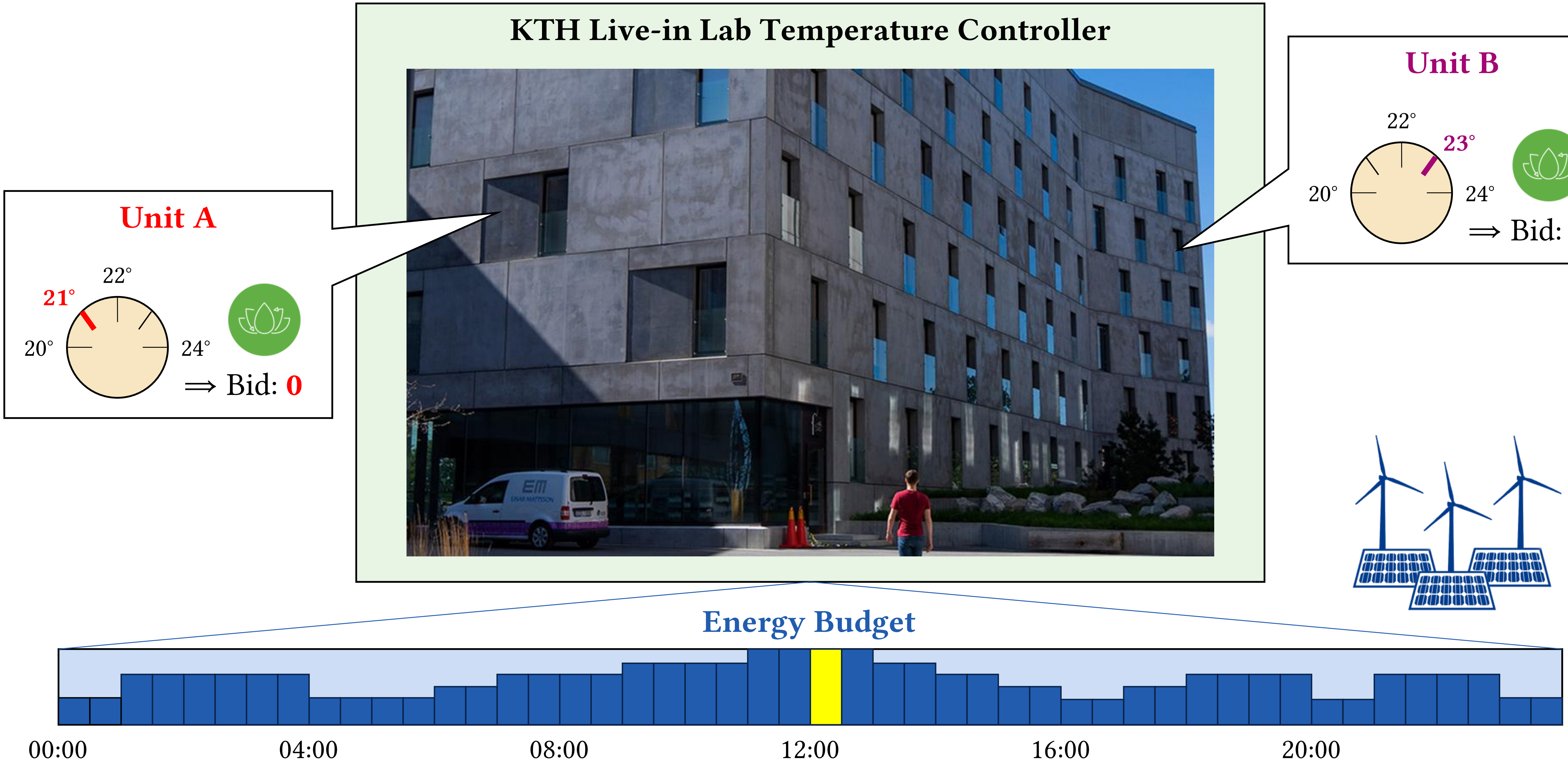
- Building operation: 28% of global CO_2 emissions
- <u>Specifically space heating:</u> 10% of global CO_2 emissions

  $\Longrightarrow$ *huge impact of sustainable heating behaviors*

(source: [International Energy Agency]())

Farjadnia, M., **Elokda, E.,** Fontan, A., Molinari, M. Sustainable Heating with Karma: A Simulation Study of the KTH Live-In Lab. *CPHS* (submitted, 2026)

Fair & Sustainable Heating with Karma



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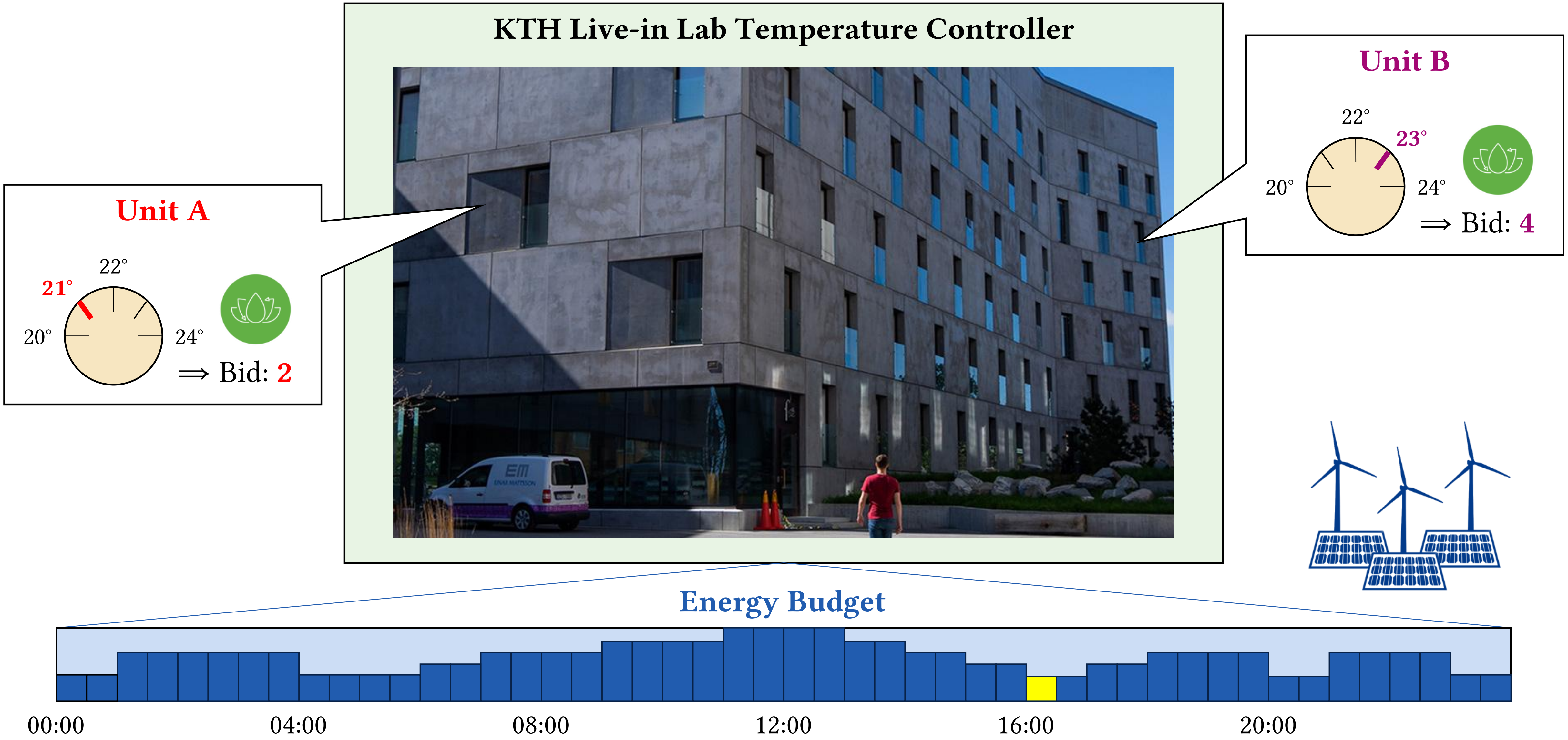
Fair & Sustainable Heating with **Karma**



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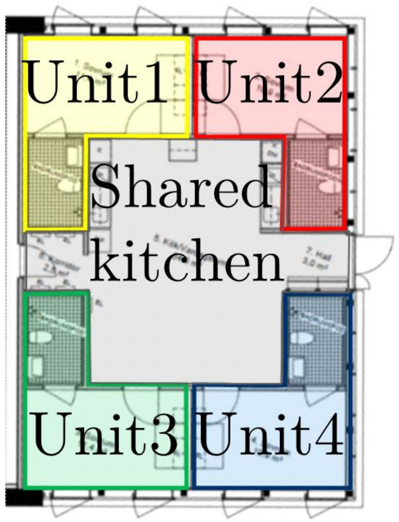
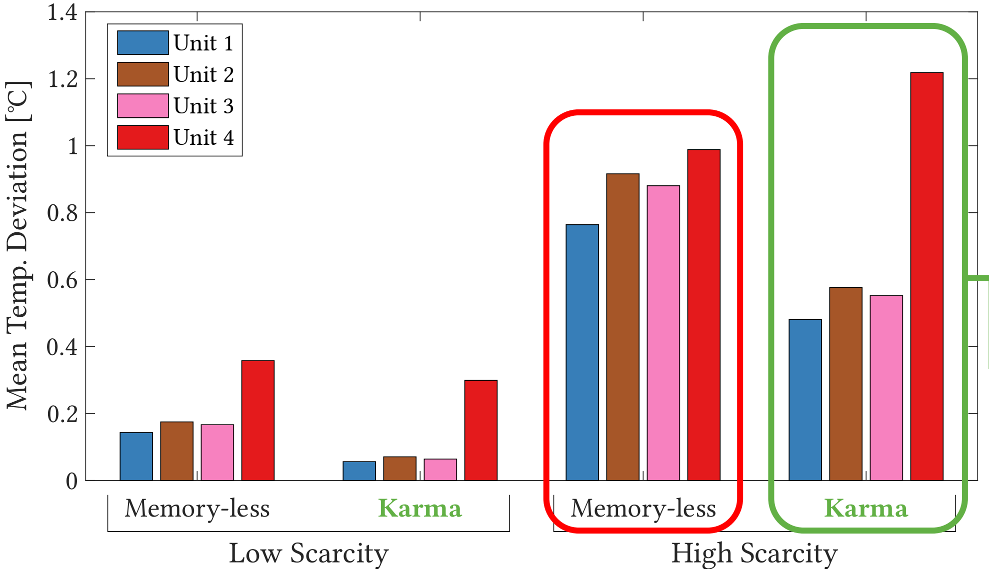
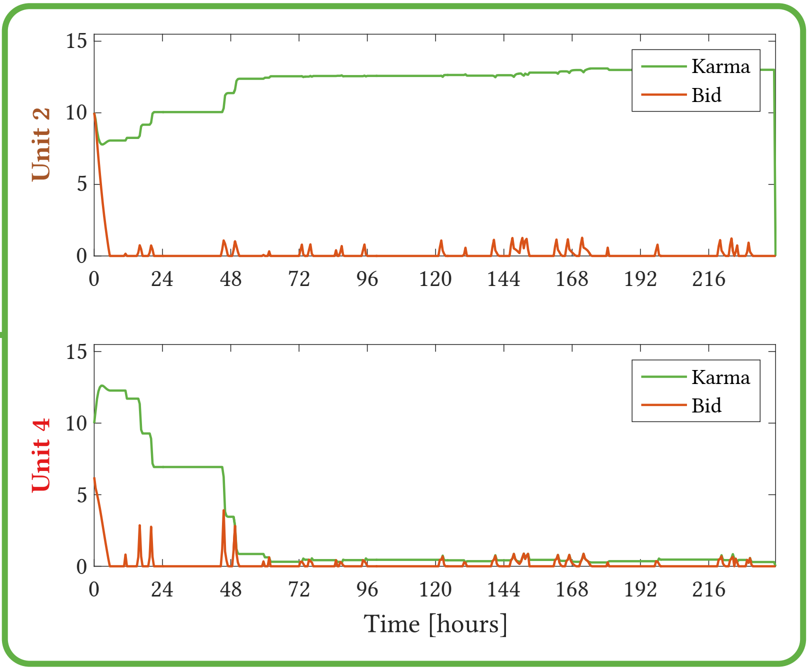
Simulation results with *Testbed KTH* digital model

"Rogue" Unit 4: keeps on opening windows







no karma: *Unit 4 "unsustainable behavior" affects all else*

karma: *Unit 4 runs out of karma*

$\Longrightarrow$ *bears consequences of "unsustainable behavior"*

Farjadnia, M., **Elokda, E.,** Fontan, A., Molinari, M. Sustainable Heating with Karma: A Simulation Study of the KTH Live-In Lab. *CPHS* (submitted, 2026)

Big Empirical Research Question:

To what extent can we expect / rely on *<u>active human engagement / decision making</u>*?

Stay tuned...

Conclusion: “Closed-Loop” **Karma Economies**

“Closed-Loop” Karma Economy

Resource Allocation Problem

outcomes
karma transfers

Karma-based Allocation Scheme

future
entitlements

- ➢ Repetitive and dynamic
- ➢ Long-run fairness / efficiency / social welfare
- ➢ Trust + fairness + efficiency compatible

More on **karma:**
elokda.info/karma

Outlook: Controllers in “Closed-Loop” w. Society

Societal Phenomenon
(e.g., opinions)

Controller
(e.g., recommender)

- ➢ Embrace dynamic & evolving nature
- ➢ Optimize long-term goals
- ➢ Alleviate trade-offs and dilemma

Prospective CDC Workshop:

Workshop on Dynamics in Societal Systems: A Hero or a Villain?

Thank you!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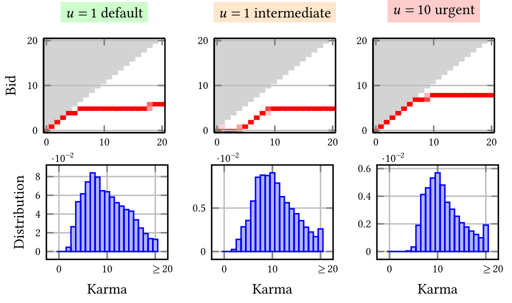
Institute for Dynamic Systems and Control IDSC Institut für Dynamische Systeme und Regelungstechnik
NCCR Automation

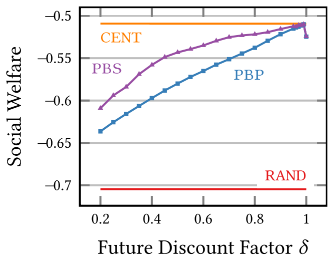
Dynamic Population Game (DPG) Model

- **Elokda, E.** *et al.* Dynamic Population Games: A Tractable Intersection of Mean-Field Games and Population Games. *IEEE Control Systems Letters* (2024)
- **Elokda, E.** *et al.* A Self-Contained Karma Economy for the Dynamic Allocation of Common Resources. *Dynamic Games and Applications* (2023)

Theorem: Reduction of Stationary Nash Equilibrium (SNE) to Nash Equilibrium (NE)

Theorem: Existence of Karma SNE



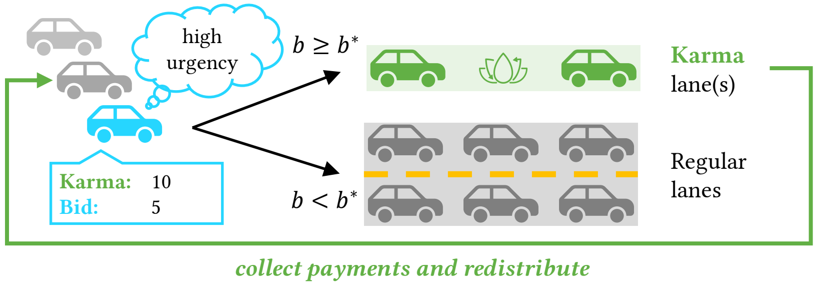


Applications: Traffic Management, EV Charging, ...

- **Elokda, E.** *et al.* CARMA: Fair and Efficient Bottleneck Congestion Management via Non-Tradable Karma Credits. *Transportation Science* (2024)
- **Elokda, E.** *et al.* Flexible Electric Vehicle Charging with Karma. *CDC* (submitted, 2026)

Theorem: Equity with respect to income

Theorem: Strong Pareto improvement over no control



Karma

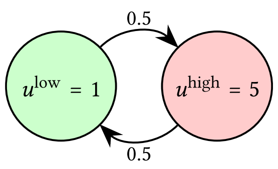
“Today me, tomorrow thee”

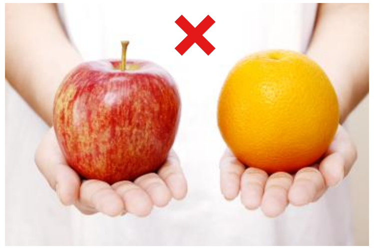
What is *Fairness* and *Efficiency*?

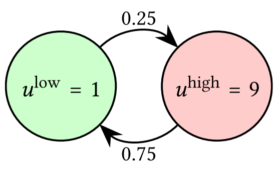
- **Elokda, E.** *et al.* A Vision for Trustworthy, Fair, and Efficient Socio-Technical Control with Karma Economies. *Annual Reviews in Control* (2025)
- Shilov, I., **Elokda, E.** *et al.* Welfare and Cost Aggregation for Multi-Agent Control: When to Choose Which Social Cost Function, and Why?. *IEEE Open Journal of Control Systems* (2025)

Theorem: Maximum Long-run Nash Welfare (MLNW) is Karma Equilibrium (KE)

Theorem: KE is MLNW

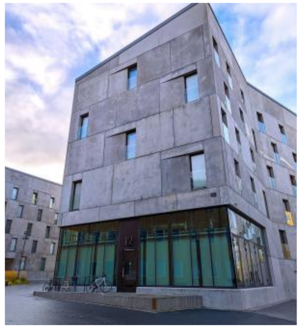


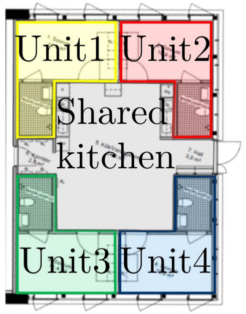


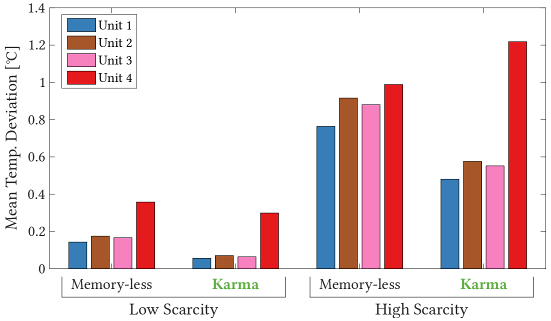


Human-in-the-loop Experiments

- **Elokda, E.** *et al.* Dynamic Allocation with Karma: An Experimental Study. *Journal of Economic Behavior and Organization* (revision pending, 2026)
- Farjadnia, M., **Elokda, E.** *et al.* Sustainable Heating with Karma: A Simulation Study of the KTH Live-In Lab. *CPHS* (submitted, 2026)





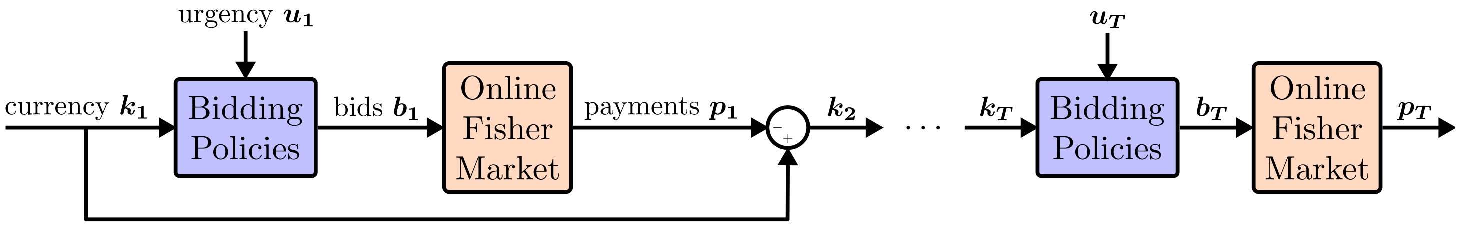


Supplementary Material

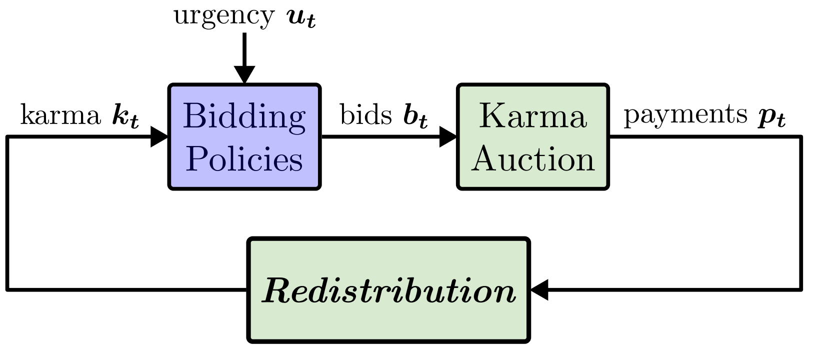
Related Mechanisms

Mechanism	Repetition?	Private dynamics?	Infinite repetition?	Connection to NW?	Notes
Divisible resources: **CE of Fisher market** (Eisenberg, 1961)	✗	✗	✗	Max (for concave homogeneous payoffs, e.g., linear)	
Indivisible resources: **CE of pseudomarket** (Hylland, Zeckhauser, 1979)	✗	✗	✗	Non-unit demand: max (in probabilities) *Unit demand: not always*	
Linking mechanism (Jackson, Sonnenschein, 2007)	✓	✗	✗	Any Pareto efficient allocation possible (including max NW)	
Online/repeated Fisher market (e.g., Gorokh, Banerjee, Iyer, 2021)	✓	✓	✗	Not studied	Show bounds w.r.t. "ideal utilities": best allocation given "fair shares"
Promised utilities (Balserio, Gurkan, Sun, 2019)	✓	✗	✓	Max utilitarianism	
Trading favors (Möbius, 2001)	✓	~	✓	Not studied	2 players, binary dynamics: available to grant favor or not
Trading chips (Olszewski, Safronov, 2018)	✓	✗	✓	Max utilitarianism	Similar to promised utilities
Food Banks Choice System (Prendergast, 2022)	✓	✓	✓	Not studied	Modelled as static CE
Karma Economies	✓	✓	✓	Max (under Nash balance)	

Related Mechanisms



(a) “Open-loop” online Fisher market. Currency is spent over a finite time T.



(b) “Closed-loop” karma economy. Karma is spent *and* gained sustainably over infinite time.

MNW in Infinitely Repeated Allocation vs. Probabilistic Pseudomarket
(non-unit demand case, resources with capacity 1)

Infinitely Repeated Allocation

$$\max_{\boldsymbol{\psi}} \sum_i \log(\bar{r}_i)$$

$$\text{s.t.:} \quad \sum_i \sum_{u_i} \sigma_i[u_i]\, \psi_{i,j}[u_i] \le 1, \qquad \forall j$$

$$0 \le \psi_{i,j}[u_i] \le 1, \qquad \forall i, u_i, j$$

KE not necessarily!

Probabilistic Pseudomarket

$$\max_{\boldsymbol{\psi}} \sum_i \log(\bar{r}_i)$$

$$\text{s.t.:} \quad \sum_i \psi_{i,j} \le 1, \qquad \forall j$$

$$0 \le \psi_{i,j}, \qquad \forall i, j$$

CE maximizes NW

MLNW in Infinitely Repeated Allocation vs. Probabilistic Pseudomarket
(unit demand/assignment case)

Infinitely Repeated Allocation

$$\max_{\boldsymbol{\psi}} \sum_i \log(\bar{r}_i)$$

$$\text{s.t.:} \quad \sum_i \sum_{u_i} \sigma_i[u_i]\, \psi_{i,j}[u_i] \leq 1, \qquad \forall j$$

$$0 \leq \sum_j \psi_{i,j}[u_i] \leq 1, \qquad \forall i, u_i$$

KE also!

Probabilistic Pseudomarket

$$\max_{\boldsymbol{\psi}} \sum_i \log(\bar{r}_i)$$

$$\text{s.t.:} \quad \sum_i \psi_{i,j} \leq 1, \qquad \forall j$$

$$0 \leq \sum_j \psi_{i,j} \leq 1, \qquad \forall i$$

CE need not maximize NW!